VWO WISKUNDE B 2019 TIJDVAK 2

Opgave 1:

$P(p\,, p(p-3)^2+2)$

$A(7\,,0)$

$AP=\sqrt{(p-7)^2+(p(p-3)^2+2)^2}$

$Y_1=\sqrt{(x-7)^2+(x(x-3)^2+2)^2}$ optie minimum geeft $y=4{,}35$

Opgave 2:

$y=\sin(2t)-\sin(t)=0$

$\sin(2t)=\sin(t)$

$2t=t+k\cdot 2\pi \quad \vee \quad 2t=\pi-t+k\cdot 2\pi$

$t=0+k\cdot 2\pi \quad \vee \quad 3t=\pi+k\cdot 2\pi$

$t=0+k\cdot 2\pi \quad \vee \quad t=\frac{1}{3}\pi+k\cdot\frac{2}{3}\pi$

$t=0 \quad \vee \quad t=\frac{1}{3}\pi \quad \vee \quad t=\pi \quad \vee \quad t=\frac{5}{3}\pi \quad \vee \quad t=2\pi$

$x\left(\frac{1}{3}\pi\right)=-\frac{1}{2}-\frac{1}{2}\sqrt{3}$

Opgave 3:

$\begin{cases} x'(t)=-2\sin(2t)-2\cos(2t) \\ y'(t)=2\cos(2t)-\cos(t) \end{cases}$

$t=0 \quad \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix}=\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$t=\pi \quad \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix}=\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$\cos\varphi=\dfrac{\begin{pmatrix} -2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} -2 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right|\cdot\left|\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right|}=\dfrac{7}{\sqrt{5}\cdot\sqrt{13}}=\dfrac{7}{\sqrt{65}}$

$\varphi=29{,}7°$

Opgave 4:

De knik van de grafiek zit bij $x_A=2$

als $x<2$ geldt: $f(x)=(-x+2)\left(\frac{1}{2}x+2\right)+1$

$=-\frac{1}{2}x^2-x+4+1$

$f'(x)=-x-1$

$\lim\limits_{x\uparrow 2} f'(x)=\lim\limits_{x\uparrow 2} -x-1=-2-1=-3$

$y=-3x+b$ door $(2\,,1)$

$1=-6+b$

$b=7$

$y=-3x+7$

Opgave 5:

$$\begin{cases} 20 \cdot 116^m = C \\ 30 \cdot 40^m = C \end{cases}$$

$$20 \cdot 116^m = 30 \cdot 40^m$$

$$\frac{116^m}{40^m} = \frac{30}{20}$$

$$\left(\frac{116}{40}\right)^m = 1{,}5$$

$$2{,}9^m = 1{,}5$$

$$m = {}^{2,9}\log(1{,}5) = 0{,}38$$

$$C = 122$$

Opgave 6:

$$N = c \cdot V$$

$$6 = c \cdot 20$$

$$c = 0{,}3$$

$$N = 0{,}3V$$

$$V \cdot T^{0,25} = 150$$

$$T^{0,25} = \frac{150}{V}$$

$$T = \left(\frac{150}{V}\right)^4$$

$$d = \frac{T}{T+2} = \frac{\left(\frac{150}{V}\right)^4}{\left(\frac{150}{V}\right)^4+2} = \frac{\frac{150^4}{V^4}}{\frac{150^4}{V^4}+2} = \frac{\frac{150^4}{V^4}}{\frac{150^4}{V^4}+\frac{2V^4}{V^4}} = \frac{\frac{150^4}{V^4}}{\frac{150^4+2V^4}{V^4}}$$

$$= \frac{150^4}{V^4} \cdot \frac{V^4}{150^4+2V^4} = \frac{150^4}{150^4+2V^4} = \frac{1}{1+\frac{2}{150^4} \cdot V^4}$$

$$A = 1440 \cdot N \cdot d = 1440 \cdot 0{,}3V \cdot \frac{1}{\frac{2}{150^4} \cdot V^4+1} = \frac{432V}{\frac{2}{150^4}V^4+1}$$

Opgave 7:

$$A' = \frac{\left(\frac{2}{150^4} \cdot V^4+1\right) \cdot 432 - 432V \cdot \frac{8}{150^4} \cdot V^3}{\left(\frac{2}{150^4}V^4+1\right)^2} = 0$$

$$\left(\frac{2}{150^4}V^4 + 1\right) \cdot 432 - 432V \cdot \frac{8}{150^4}V^3 = 0$$

$$\frac{864}{150^4}V^4 + 432 - \frac{3456}{150^4}V^4 = 0$$

$$-\frac{2592}{150^4}V^4 = -432$$

$$V^4 = 84375000$$

$$V = 95{,}8$$

Opgave 8:

$A(p) = \int_p^{\pi-p} 2\sin(x)dx = [-2\cos(x)]_p^{\pi-p} = -2\cos(\pi - p) + 2\cos(p)$

$= 2\cos(p) + 2\cos(p) = 4\cos(p)$

Opgave 9:

$Opp(W) = (\pi - 2p) \cdot 2\sin(p)$

$(\pi - 2p) \cdot 2\sin(p) = 2\cos(p)$

$Y_1 = (\pi - 2x) \cdot 2\sin(x)$ en $Y_2 = 2\cos(x)$ optie intsect geeft: $x = 0{,}41$

dus $p = 0{,}41$

Opgave 10:

V.A.: $e^x - 10 = 0$

$e^x = 10$

$x = \ln(10)$

H.A.: $\lim_{x\to-\infty} \frac{e^{2x}-1000}{e^x-10} = \lim_{x\to-\infty} \frac{0-1000}{0-10} = 100$ dus $y = 100$

$B(\ln(10), 100)$

$A(0\,, 100)$

$\frac{e^{2x}-1000}{e^x-10} = 100$

$e^{2x} - 1000 = 100e^x - 1000$

$e^{2x} = 100e^x$

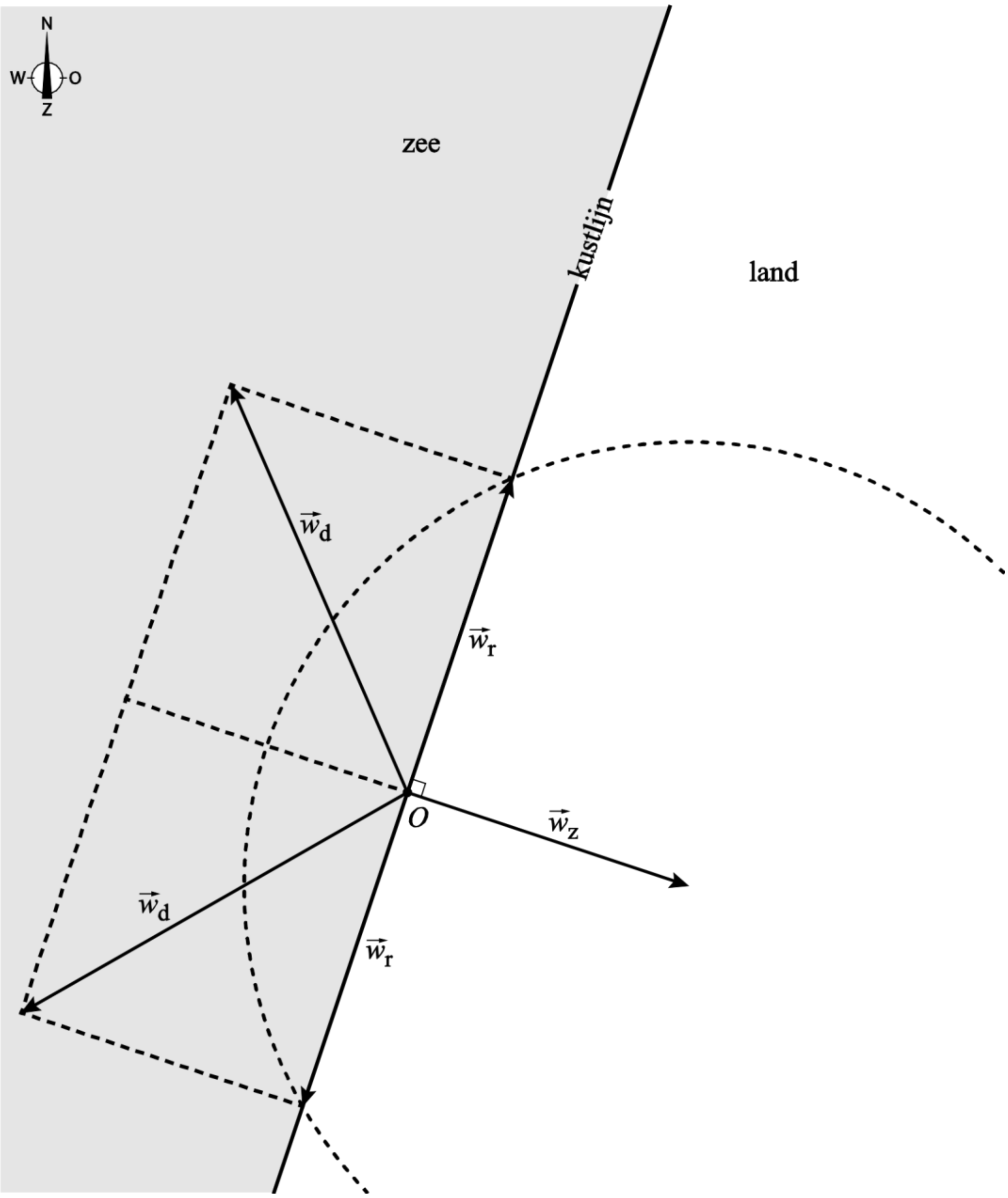
$e^x = 0 \quad \vee \quad e^x = 100$

k.n. $\quad x = \ln(100) = \ln(10)^2 = 2\ln(10)$

$\frac{x_A+x_C}{2} = \frac{0+2\ln(10)}{2} = \ln(10) = x_B$ dus B is het midden van AC

Opgave 11:

Teken een cirkel met straal 6 en als middelpunt het eindpunt van $\overrightarrow{W_z}$
Deze cirkel snijdt de kustlijn in twee punten, die de eindpunten zijn van de vectoren $\overrightarrow{W_r}$ (met O als beginpunt)
Teken $\overrightarrow{W_d}$ met de parallellogramconstructie zo dat $\overrightarrow{W_d} = \overrightarrow{W_r} - \overrightarrow{W_z}$



Opgave 12:

$$\overrightarrow{W_z} = \begin{pmatrix} 3\cos(-30°) \\ 3\sin(-30°) \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2}\sqrt{3} \\ -1\frac{1}{2} \end{pmatrix}$$

$$\overrightarrow{W_d} = \begin{pmatrix} 5\cos(-135°) \\ 5\sin(-135°) \end{pmatrix} = \begin{pmatrix} -2\frac{1}{2}\sqrt{2} \\ -2\frac{1}{2}\sqrt{2} \end{pmatrix}$$

$$\overrightarrow{W_r} = \overrightarrow{W_z} + \overrightarrow{W_d} = \begin{pmatrix} 1\frac{1}{2}\sqrt{3} \\ -1\frac{1}{2} \end{pmatrix} + \begin{pmatrix} -2\frac{1}{2}\sqrt{2} \\ -2\frac{1}{2}\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2}\sqrt{3} - 2\frac{1}{2}\sqrt{2} \\ -1\frac{1}{2} - 2\frac{1}{2}\sqrt{2} \end{pmatrix}$$

$$\left|\overrightarrow{W_r}\right| = \sqrt{(1\frac{1}{2}\sqrt{3} - 2\frac{1}{2}\sqrt{2})^2 + (-1\frac{1}{2} - 2\frac{1}{2}\sqrt{2})^2} = 5{,}1$$

Opgave 13:

$f(x) = g(x+3)$

$\log(\sqrt{x}) = \log\left((x+3)\sqrt{x+3}\right) - 1$

$Y_1 = \log(\sqrt{x})$ en $Y_2 = \log((x+3)\sqrt{x+3}) - 1$

optie intsect geeft $y = -0{,}20 \quad \vee \quad y = 0{,}34$

dus $q = -0{,}20 \quad \vee \quad q = 0{,}34$

Opgave 14:

$E(p\,,0)$

$C(p\,,\log(\sqrt{p}))$

$D(p\,,\log(p\sqrt{p}) - 1)$

$$CD = \log(p\sqrt{p}) - 1 - \log(\sqrt{p}) = \log\left(\frac{p\sqrt{p}}{p}\right) - 1 = \log(p) - 1$$

$$CE = \log(\sqrt{p}) = \log(p^{\frac{1}{2}}) = \frac{1}{2}\cdot\log(p)$$

$$\frac{CD}{CE} = \frac{\log(p) - 1}{\frac{1}{2}\cdot\log(p)} = \frac{2\log(p) - 2}{\log(p)}$$

Opgave 15:

$$\lim_{p\to\infty} \frac{2\log(p) - 2}{\log(p)} = \lim_{p\to\infty} \frac{2 - \frac{2}{\log(p)}}{1} = \frac{2-0}{1} = 2$$

Opgave 16:

cirkel: $x^2 + (y-r)^2 = r^2$

snijden met de parabool geeft:

$y + (y-r)^2 = r^2$

$y + y^2 - 2ry + r^2 = r^2$

$y^2 + (1-2r)y = 0$

$y(y + 1 - 2r) = 0$

$y = 0 \quad \vee \quad y = -1 + 2r$

$-1 + 2r > 0$

$2r > 1$

$r > \frac{1}{2}$

Opgave 17:

$V + W$ wentelen rond de y-as:

$I_{V+W} = \pi \int_0^r x^2 dy = \pi \int_0^r y dy = \pi \left[\frac{1}{2}y^2\right]_0^r = \frac{1}{2}\pi r^2$

dus $I_W = \frac{1}{4}\pi r^2$

maar ook: $I_W = \frac{1}{2} \cdot I_{bol} = \frac{2}{3}\pi r^3$

$\frac{1}{4}\pi r^2 = \frac{2}{3}\pi r^3$

$r = 0 \quad \vee \quad \frac{1}{4}\pi = \frac{2}{3}\pi r$

$r = \frac{3}{8}$