VWO WISKUNDE B 2019 TIJDVAK 1

Opgave 1:
cirkel: $(x-1)^2+(y-7)^2=25$
de cirkel snijden met lijn k geeft:
$(t-1)^2+(2t-7)^2=25$
$t^2-2t+1+4t^2-28t+49=25$
$5t^2-30t+25=0$
$t^2-6t+5=0$
$(t-1)(t-5)=0$
$t=1 \quad \vee \quad t=5$
$(1,2) \qquad (5,10)$

Opgave 2:
$g(x)=3-\sqrt{2x}$
$\sqrt{2x}=3$
$2x=9$
$x=4\frac{1}{2}$ dus $A(4\frac{1}{2},0)$
$$f'(x)=-3\cdot\sin(2x)\cdot 2-\frac{1}{2\sqrt{2x}}\cdot 2$$
$$=-6\sin(2x)-\frac{1}{\sqrt{2x}}=0$$
$Y_1=-6\sin(2x)-\frac{1}{\sqrt{2x}}=0$ root geeft $x_B=4{,}74>4\frac{1}{2}$
dus punt B ligt rechts van punt A

Opgave 3:
$f_p(x)=p+\sqrt{x-p}$
$$f'_p(x)=\frac{1}{2\sqrt{x-p}}=1$$
$\sqrt{x-p}=\frac{1}{2}$
$x-p=\frac{1}{4}$
$x=p+\frac{1}{4}$
$f_p\left(p+\frac{1}{4}\right)=p+\sqrt{p+\frac{1}{4}-p}=p+\frac{1}{2}$
$y=p+\frac{1}{4}+\frac{1}{4}=p+\frac{1}{2}$
dus k raakt de grafiek van f_p

Opgave 4:
$f_p(x) = p + \sqrt{x - p}$
randpunt $x = p$ en $y = p$
$f_{p-1}(p) = p - 1 + \sqrt{p - (p - 1)}$
$= p - 1 + \sqrt{p - p + 1}$
$= p - 1 + 1 = p$
dus (p, p) ligt op de grafiek van f_{p-1}

Opgave 5:
lijn l: $y = x$
$$Opp(V) = \int_1^2 \left(1 + \sqrt{x-1} - x\right)dx$$
$$= \left[x + \tfrac{2}{3}(x-1)\sqrt{x-1} - \tfrac{1}{2}x^2\right]_1^2$$
$$= 2 + \tfrac{2}{3} - 2 - \left(1 - \tfrac{1}{2}\right) = \tfrac{1}{6}$$

Opgave 6:
$BC = \sqrt{20^2 + 7^2} = \sqrt{449}$
$F_k = 0{,}6 \cdot \left(\sqrt{449} - 8\right) = 7{,}9$ kN

Opgave 7:
$L = \sqrt{x^2 + 49}$
$\cos\alpha = \dfrac{x}{L}$
$F_{kv} = 2 \cdot F_k \cdot \cos\alpha = 2 \cdot 0{,}6\left(\sqrt{x^2+49} - 8\right) \cdot \dfrac{x}{\sqrt{x^2+49}} = 1{,}8$
$Y_1 = 1{,}2 \cdot (\sqrt{x^2 + 491} - 8) \cdot \dfrac{x}{\sqrt{x^2+49}}$
$Y_2 = 1{,}8$
intsect geeft $x = 7$
dus de hoogte is 13 m

Opgave 8:
$f'(x) = \ln(x)$
$f(x) = g(x)$ dus $x \cdot \ln(x) - x + 1 = \ln(x)$
$x \cdot \ln(x) - \ln(x) = x - 1$
$(x - 1) \cdot \ln(x) = x - 1$
$x - 1 = 0 \quad \vee \quad \ln(x) = 1$
$x = 1 \quad \vee \quad x = e$

Opgave 9:

$$\int_p^{2p} \ln(x)dx = [x \cdot \ln(x) - x]_p^{2p}$$

$$= 2p \cdot \ln(2p) - 2p - (p \cdot \ln(p) - p)$$

$$= 2p \cdot \ln(2p) - p \cdot \ln(p) - p = 0$$

$p(2\ln(2p) - \ln(p) - 1) = 0$

$p = 0 \quad \vee \quad 2\ln(2p) - \ln(p) = 1$

k.n. $\quad \ln(4p^2) - \ln(p) = 1$

$\ln(4p) = 1$

$4p = e$

$p = \frac{1}{4}e$

Opgave 10:

$$\frac{\cos(x)}{-\sin^2(x)} = \sqrt{2}$$

$\cos(x) = -\sqrt{2} \cdot \sin^2(x)$

$\cos(x) = -\sqrt{2} \cdot (1 - \cos^2(x))$

$\sqrt{2} \cdot \cos^2(x) - \cos(x) - \sqrt{2} = 0$

$\sqrt{2} \cdot p^2 - p - \sqrt{2} = 0$

$$p = \frac{1 \pm \sqrt{9}}{2\sqrt{2}}$$

$$p = \frac{-2}{2\sqrt{2}} = -\frac{1}{2}\sqrt{2} \quad \vee \quad p = \frac{4}{2\sqrt{2}} = \sqrt{2}$$

$$\cos(x) = -\frac{1}{2}\sqrt{2} \quad \vee \quad \cos(x) = \sqrt{2}$$

$$x = \frac{3}{4}\pi \quad \vee \quad x = 1\frac{1}{4}\pi$$

Opgave 11:

$$f_p(x) = \frac{\cos(x)}{p - \sin^2(x)}$$

perforatie als $\cos(x) = 0 \quad \wedge \quad p - \sin^2(x) = 0$

$x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{1}{2}\pi + k \cdot 2\pi$

voor die x-waarden geldt: $\sin(x) = 1 \quad \vee \quad \sin(x) = -1$

$p - \sin^2(x) = p - 1 = 0$

$p = 1$

$$f_1(x) = \frac{\cos(x)}{1 - \sin^2(x)} = \frac{\cos(x)}{\cos^2(x)} = \frac{1}{\cos(x)}$$

$\lim_{x \to \frac{1}{2}\pi} \frac{1}{\cos(x)}$ = bestaat niet , dus er is geen perforatie

Opgave 12:

$P = \left(0 , \frac{1}{p}\right) \quad Q = \left(\pi , \frac{-1}{p}\right) \quad R = (2\pi , \frac{1}{p})$

$\overrightarrow{PQ} = \begin{pmatrix} \pi \\ \frac{-2}{p} \end{pmatrix}$

$\overrightarrow{QR} = \begin{pmatrix} \pi \\ \frac{2}{p} \end{pmatrix}$

$\overrightarrow{PQ} \cdot \overrightarrow{QR} = \begin{pmatrix} \pi \\ \frac{-2}{p} \end{pmatrix} \cdot \begin{pmatrix} \pi \\ \frac{2}{p} \end{pmatrix} = \pi^2 - \frac{4}{p^2} = 0$

$\frac{4}{p^2} = \pi^2$

$p^2 = \frac{4}{\pi^2}$

$p = \frac{2}{\pi} \quad \vee \quad p = -\frac{2}{\pi}$

Opgave 13:

A , B en P vormen geen driehoek als punt P op lijn AB ligt

lijn AB: $\frac{x}{40} + \frac{y}{10} = 1$

$x + 4y = 40$

P invullen in lijn AB geeft:

$18 + 5t + 4(30 - 3t) = 40$

$18 + 5t + 120 - 12t = 40$

$-7t = -98$

$t = 14$

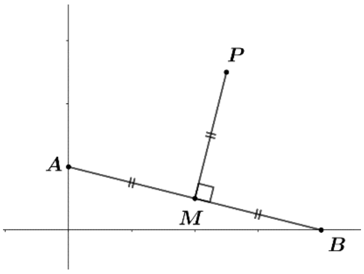
$P(88, -12)$

Opgave 14:

$\overrightarrow{MA} = \begin{pmatrix} -20 \\ 5 \end{pmatrix}$

$\overrightarrow{MP} = \overrightarrow{MA}_r = \begin{pmatrix} 5 \\ 20 \end{pmatrix}$

$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \begin{pmatrix} 20 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 20 \end{pmatrix} = \begin{pmatrix} 25 \\ 25 \end{pmatrix}$



$\begin{cases} 18+5t=25 \\ 30-3t=25 \end{cases}$ dus $\begin{cases} 5t=7 \\ -3t=-5 \end{cases}$ dus $\begin{cases} t=\frac{7}{5} \\ t=\frac{5}{3} \end{cases}$ dus dit is niet mogelijk

Opgave 15:

$m=\frac{1}{2}(a+b)$

$A=\pi\cdot\left(\sqrt{m}\right)^2=\pi\cdot m=\pi\cdot(\frac{1}{2}a+\frac{1}{2}b)$

$h=b-a$

$V=(b-a)\cdot\pi\cdot\left(\frac{1}{2}a+\frac{1}{2}b\right)=(b-a)\cdot\pi\cdot\frac{1}{2}(b+a)=\frac{1}{2}\pi(b^2-a^2)$

$Inh=\pi\cdot\int_a^b(\sqrt{x})^2dx=\pi\cdot\int_a^b xdx=\pi\cdot\left[\frac{1}{2}x^2\right]_a^b=\pi\left(\frac{1}{2}b^2-\frac{1}{2}a^2\right)=\frac{1}{2}\pi(b^2-a^2)$