# VWO WISKUNDE B 2018 TIJDVAK 2

**Opgave 1:**

$f(x) = h(x)$

$\frac{-2 + 2\sqrt{x+1}}{x} = \frac{2}{1 + \sqrt{x+1}}$

$2x = (1 + \sqrt{x+1})(-2 + 2\sqrt{x+1})$

$2x = -2 + 2\sqrt{x+1} - 2\sqrt{x+1} + 2(x+1)$

$2x = -2 + 2x + 2$

$2x = 2x$ (klopt voor $x \neq 0$)

**Opgave 2:**

$g(x) = \frac{4x^2+x}{x} = 4x + 1$ dus lijn $k$: $y = 4x + 1$

$h(x) = \frac{2}{\left(1 + \sqrt{x+1}\right)} = 2\left(1 + \sqrt{x+1}\right)^{-1}$

$h'(x) = -2\left(1 + \sqrt{x+1}\right)^{-2} \cdot \frac{1}{2\sqrt{x+1}} = \frac{-1}{(1 + \sqrt{x+1})^2 \cdot \sqrt{x+1}}$

$h'(0) = -\frac{1}{4}$

$rc_k = 4$

$h'(0) \cdot rc_k = -\frac{1}{4} \cdot 4 = -1$ dus $k$ en $h$ staan loodrecht op elkaar

**Opgave 3:**

$\frac{4}{3}\pi r^3 = 27$

$r^3 = \frac{81}{4\pi}$

$r = \sqrt[3]{\frac{81}{4\pi}}$

$A = 4\pi r^2 = 4\pi \left(\sqrt[3]{\frac{81}{4\pi}}\right)^2$

$\frac{A}{V} = \frac{4\pi\left(\sqrt[3]{\frac{81}{4\pi}}\right)^2}{27} = 1{,}61$

**Opgave 4:**

$\pi \int_{-1,5}^{a} x^2 dy = \pi \int_{-1,5}^{a} (2{,}25 - y^2) dy = \pi \left[2{,}25y - \frac{1}{3}y^3\right]_{-1,5}^{a}$

$= \pi \left(2{,}25a - \frac{1}{3}a^3 - -2{,}25\right) = \pi \left(2{,}25a - \frac{1}{3}a^3 + 2{,}25\right) = \frac{207}{50}\pi$

$y_1 = \pi(2{,}25x - \frac{1}{3}x^3 + 2{,}25)$ en $y_2 = \frac{207}{50}\pi$

intsect geeft $x = 0{,}98$ dus $a = 0{,}98$

**Opgave 5:**

$t = 0 \quad r = 1{,}5 \quad V(0) = \frac{4}{3}\pi \cdot 1{,}5^3 = 4\frac{1}{2}\pi$

$t = 10 \quad V(10) = 2\frac{1}{4}\pi = \frac{4}{3}\pi r^3$

$r(10) = 1{,}19055$

$rc = \frac{1{,}19055 - 1{,}5}{10} = -0{,}030944$

$r = -0{,}030944t + 15 = 0$

$t = 48{,}47$ dus na 49 minuten

**Opgave 6:**

$f_a(x) + f_{\frac{1}{a}}(x) = 2 \cdot f_1(x)$

$$\begin{aligned} f_a(x) + f_{\frac{1}{a}}(x) &= x - xln(ax) + x - xln(\tfrac{1}{a}x) \\ &= 2x - x\left(\ln(ax) + \ln\left(\tfrac{1}{a}x\right)\right) \\ &= 2x - xln(x^2) \\ &= 2x - 2xln(x) \\ &= 2\big(x - xln(x)\big) = 2 \cdot f_1(x) \end{aligned}$$

**Opgave 7:**

$x - xln(ax) = 0$

$x(1 - \ln(ax)) = 0$

$x = 0 \quad \vee \quad \ln(ax) = 1$

$ax = e$

$x_S = \frac{e}{a}$

$f'_a(x) = 1 - \left(1 \cdot \ln(ax) + x \cdot \frac{1}{ax} \cdot a\right) = 1 - \ln(ax) - 1 = -\ln(ax) = 0$

$ax = e^0 = 1$

$x_T = \frac{1}{a}$

$\frac{x_S}{x_T} = \frac{\frac{e}{a}}{\frac{1}{a}} = e$

**Opgave 8:**

Omdat $\angle B = 90°$ geldt dat $CP$ de middellijn van de cirkel is (Thales).
Dus het middelpunt $M$ van de cirkel is het midden van $CP$.
dus $M = (-1, -\frac{1}{2})$.

$r = CM = \sqrt{3^2 + (3\frac{1}{2})^2} = \sqrt{21\frac{1}{4}}$

cirkel: $(x+1)^2 + (y + \frac{1}{2})^2 = 21\frac{1}{4}$

**Opgave 9:**

lijn $PD$: $rc = \frac{-11}{2} = -5\frac{1}{2}$

$y = -5\frac{1}{2}x + b$ door (2,3)

$3 = -11 + b$

$b = 14$

$y = -5\frac{1}{2}x + 14$

lijn $CQ$: $rc = \frac{2}{11}$

$y = \frac{2}{11}x + b$ door $(-4, -4)$

$-4 = -\frac{8}{11} + b$

$b = -3\frac{3}{11}$

$y = \frac{2}{11}x - 3\frac{3}{11}$

$-5\frac{1}{2}x + 14 = \frac{2}{11}x - 3\frac{3}{11}$

$x = 3\frac{1}{25}$

$y = -2\frac{18}{25}$

dus $Q = \left(3\frac{1}{25}, -2\frac{18}{25}\right)$

**Opgave 10:**

$Opp\Delta CDQ = \frac{1}{3} \cdot Opp(ABCD)$

$\frac{1}{2} \cdot CD \cdot h = \frac{1}{3} \cdot CD^2$

$h = \frac{2}{3} \cdot CD$

$DQ: PQ = 2: 1$

$$\overrightarrow{OQ} = \overrightarrow{OD} + \frac{2}{3} \cdot \overrightarrow{QP} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix} -2 \\ 11 \end{pmatrix} = \begin{pmatrix} 2\frac{2}{3} \\ -\frac{2}{3} \end{pmatrix}$$

dus $Q = \left(2\frac{2}{3}, -\frac{2}{3}\right)$

**Opgave 11:**

lijn $OP$: $y = px$

$$Opp(V) = \int_0^p (px - x^2)dx = \left[\frac{1}{2}px^2 - \frac{1}{3}x^3\right]_0^p = \frac{1}{2}p^3 - \frac{1}{3}p^3 = \frac{1}{6}p^3$$

$f'(x) = 2x$

$f'(p) = 2p$

raaklijn: $y = 2px + b$ door $(p, p^2)$

$p^2 = 2p^2 + b$

$b = -p^2$

$$y = 2px - p^2$$

snijpunt $x$-as: $2px - p^2 = 0$

$2px = p^2$

$x = \frac{1}{2}p$

$Opp\Delta OAP = \frac{1}{2} \cdot \frac{1}{2}p \cdot p^2 = \frac{1}{4}p^3$

$1\frac{1}{2} \cdot Opp(V) = 1\frac{1}{2} \cdot \frac{1}{6}p^3 = \frac{1}{4}p^3 = Opp\Delta OAP$

**Opgave 12:**

$x_P(a) = cosa \cdot sin2a$

$x_P(\pi - a) = \cos(\pi - a) \cdot sin2(\pi - a)$

$= \cos(\pi - a) \cdot \sin(2\pi - 2a)$

$= \cos(\pi - a) \cdot \sin(-2a)$

$= -\cos a \cdot -sin2a$

$= cosa \cdot sin2a = x_P(a)$

Dus het lijnstuk tussen de punten $P_a$ en $P_{\pi - a}$ is verticaal

**Opgave 13:**

$d(P, x - as) = |cost|$

$d(P, y - as) = |cost \cdot sin2t|$

$|cost| = 2|cost \cdot sin2t|$

$|cost| = 0 \quad \vee \quad 1 = 2|sin2t|$

$cost = 0$ $\qquad$ $|sin2t| = \frac{1}{2}$

$t = \frac{1}{2}\pi + k \cdot 2\pi$ $\qquad$ $2t = \frac{1}{6}\pi + k \cdot \pi \quad \vee \quad 2t = \frac{5}{6}\pi + k \cdot \pi$

vervalt $\qquad$ $t = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi \quad \vee \quad t = \frac{5}{12}\pi + k \cdot \frac{1}{2}\pi$

dus $t = \frac{11}{12}\pi$

**Opgave 14:**

$x\left(\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2} \cdot -1 = \frac{1}{2}\sqrt{2}$

$y\left(\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2}$

$\overrightarrow{OP_t} = \begin{pmatrix} \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \end{pmatrix}$

$x'(t) = -sint \cdot sin2t + cost \cdot 2cos2t$

$y'(t) = -sint$

$x'\left(\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2} \cdot -1 + -\frac{1}{2}\sqrt{2} \cdot 0 = \frac{1}{2}\sqrt{2}$

$y'\left(\frac{3}{4}\pi\right) = -\frac{1}{2}\sqrt{2}$

$\overrightarrow{v(\frac{3}{4}\pi)} = \begin{pmatrix} \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \end{pmatrix} = \overrightarrow{OP_t}$

**Opgave 15:**

cirkel: $(x-3)^2 + (y-2)^2 = 5$
lijn $AC$: $y = -x + 4$
$(x-3)^2 + (-x+2)^2 = 5$
$x^2 - 6x + 9 + x^2 - 4x + 4 = 5$
$2x^2 - 10x + 8 = 0$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 4 \quad \vee \quad x = 1$
$y = 3$
Het midden tussen het punt $(0,4)$ en $(2,2)$ is het punt $(1,3)$
Dus $F$ is het midden van $CS$

**Opgave 16:**

$\overrightarrow{MF} \cdot \overrightarrow{MB} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = -2 + 2 = 0$ dus $\angle BMF = 90°$ dus cirkelsector $BMF$ is een kwart cirkel
$\overrightarrow{MG} \cdot \overrightarrow{MA} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = -2 + 2 = 0$ dus $\angle AMG = 90°$ dus cirkelsector $AMG$ is een kwart cirkel
Dus de oppervlakte van de twee cirkelsectoren is gelijk aan de helft van de oppervlakte van de cirkel.