VWO WISKUNDE B 2018 TIJDVAK 1

Opgave 1:

$x = 1 - t^2 = 0$
$t = 1 \quad \vee \quad t = -1$
punt A punt O
$x'(t) = -2t$ dus $x'(1) = -2$
$y'(t) = 2(1 + t)$ dus $y'(1) = 4$
$V_A = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$

Opgave 2:

$$(x + y)^2 = (1 - t^2 + (1 + t)^2)^2$$
$$= (1 - t^2 + 1 + 2t + t^2)^2$$
$$= (2 + 2t)^2$$
$$= (2(1 + t))^2$$
$$= 4(1 + t)^2 = 4y$$

Opgave 3:

$f'_a(x) = e^{ax} + axe^{ax} = 0$
$(1 + ax)e^{ax} = 0$
$ax = -1 \quad \vee \quad e^{ax} = 0$
$x = -\frac{1}{a}$ k.n.
$f_a\left(-\frac{1}{a}\right) = -\frac{1}{a} \cdot e^{-1} = \frac{-1}{ae}$
top $\left(-\frac{1}{a}, \frac{-1}{ae}\right)$
$y = \frac{1}{e} \cdot -\frac{1}{a} = \frac{-1}{ae}$ dus de top ligt op de lijn $y = \frac{1}{e} \cdot x$

Opgave 4:

$$F_a(x) = \frac{1}{a} \cdot x \cdot e^{ax} - \frac{1}{a^2} e^{ax}$$
$$F'_a(x) = \frac{1}{a} \cdot e^{ax} + \frac{1}{a} \cdot x \cdot ae^{ax} - \frac{1}{a^2} e^{ax} \cdot a$$
$$= \frac{1}{a} \cdot e^{ax} + xe^{ax} - \frac{1}{a} \cdot e^{ax}$$
$= xe^{ax} = f_a(x)$ dus F_a is een primitieve van f_a

Opgave 5:

$$xe^x = \frac{1}{e} \cdot x$$
$$x = 0 \quad \vee \quad e^x = \frac{1}{e}$$
$$x = -1$$
$$Opp = \int_{-1}^{0} \frac{1}{e} \cdot x - xe^x dx = \left[\frac{1}{2e}x^2 - (xe^x - e^x)\right]_{-1}^{0} = 0 - 0 + 1 - \left(\frac{1}{2e} + \frac{1}{e} + \frac{1}{e}\right) = 1 - \frac{5}{2e}$$

Opgave 6:

cirkel: $(x-14)^2+(y-8)^2=100$

snijpunten x-as: $(x-14)^2+64=100$

$(x-14)^2=36$

$x-14=6 \quad \vee \quad x-14=-6$

$x_B=20 \qquad x_A=8$

$6\cdot\begin{pmatrix}x_Z\\y_Z\end{pmatrix}=3\cdot\begin{pmatrix}8\\0\end{pmatrix}+1\cdot\begin{pmatrix}20\\0\end{pmatrix}+2\cdot\begin{pmatrix}14\\8\end{pmatrix}=\begin{pmatrix}72\\16\end{pmatrix}$

$\begin{pmatrix}x_Z\\y_Z\end{pmatrix}=\begin{pmatrix}12\\2\frac{2}{3}\end{pmatrix}$ dus $Z=(12\,,2\frac{2}{3})$

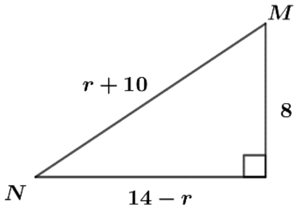
Opgave 7:

$(r+10)^2=(14-r)^2+8^2$

$r^2+20r+100=196-28r+r^2+64$

$48r=160$

$r=3\frac{1}{3}$



Opgave 8:

$f(x)=6\sin x-\cos 2x$

$f'(x)=6\cos x+2\sin 2x=0$

$6\cos x+4\sin x\cos x=0$

$4\cos x\,(1\frac{1}{2}+\sin x)=0$

$\cos x=0 \quad \vee \quad \sin x=-1\frac{1}{2}$

$x=\frac{1}{2}\pi+k\cdot 2\pi \quad \vee \quad x=-\frac{1}{2}\pi+k\cdot 2\pi$ ($\sin x=-1\frac{1}{2}$ heeft geen oplossingen)

dus $x=\frac{1}{2}\pi+k\cdot\pi$

Opgave 9:

$f\left(1\frac{1}{2}\pi+1\right)=-3{,}66$

$d=5-3{,}66=1{,}34$

Opgave 10:

$f(x)=\frac{1}{1{,}4}(e^{0{,}7x}+e^{-0{,}7x})$

$f(0)=\frac{2}{1{,}4}=1{,}4286$

$f(3)=5{,}9204$

$d=5{,}9204-1{,}4286=4{,}4918$

$l^2=4\cdot 4{,}4918^2+\frac{8\cdot 4{,}4918}{0{,}7}=132{,}04$

$l=11{,}49$

Opgave 11:

$l = 49{,}63 \quad d = 20{,}51$
$k = 0{,}21$
$f(x) = 2{,}3784(e^{0{,}21x} + e^{-0{,}21x})$
spiegel in de x-as geeft: $y = -2{,}3784(e^{0{,}21x} + e^{-0{,}21x})$
translatie $(0, b)$ geeft: $y = -2{,}3784(e^{0{,}21x} + e^{-0{,}21x}) + b$
$y(0) = -4{,}76 + b = 20{,}51$
$b = 25{,}27$
$h(x) = -2{,}38(e^{0{,}21x} + e^{-0{,}21x}) + 25{,}27$

Opgave 12:

$f(x) = \ln\sqrt{x}$
$y = \ln\sqrt{x}$
inverse: $x = \ln\sqrt{y}$
$\sqrt{y} = e^x$
$y = (e^x)^2 = e^{2x}$

Opgave 13:

$$g(x) = \frac{1}{2}e^{2x}$$
$$L = g(x) - f(x) = \frac{1}{2}e^{2x} - \ln\sqrt{x}$$
$y_1 = \frac{1}{2}e^{2x} - \ln\sqrt{x}$ optie min geeft: $L_{min} = 1{,}512$

Opgave 14:

$\ln x = 0$ dus $x = 1$
$$\lim_{x\to1}\frac{\ln\sqrt{x}}{\ln x} = \lim_{x\to1}\frac{\ln x^{\frac{1}{2}}}{\ln x} = \lim_{x\to1}\frac{\frac{1}{2}\ln x}{\ln x} = \lim_{x\to1}\frac{1}{2} = \frac{1}{2}$$
dus de perforatie is $(1\,,\frac{1}{2})$

Opgave 15:

noem de zijde van het vierkant p , dan geldt $f(1+p) = \frac{1}{1+p} = p$
$p^2 + p = 1$
$p^2 + p - 1 = 0$
$$p = \frac{-1 \pm \sqrt{5}}{2}$$
$p = -\frac{1}{2} + \frac{1}{2}\sqrt{5} \quad \vee \quad p = -\frac{1}{2} - \frac{1}{2}\sqrt{5}$ (vervalt)
dus een zijde is $-\frac{1}{2} + \frac{1}{2}\sqrt{5}$

Opgave 16:

$A(1,0)$ en $C(\cos t, \sin t)$

$AC^2 = (1 - \cos t)^2 + sin^2 t$

$BC^2 = cos^2 t + (1 - \sin t)^2$

$AC^2 = 2 \cdot BC^2$ dus $(1 - \cos t)^2 + sin^2 t = 2cos^2 t + 2(1 - \sin t)^2$

$y_1 = (1 - \cos x)^2 + sin^2 x$ en $y_2 = 2cos^2 x + 2(1 - \sin x)^2$

optie intsect geeft: $x = 0{,}93$ dus $t = 0{,}93$

Opgave 17:

$\overrightarrow{OC} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

$\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB} = \begin{pmatrix} -\cos t \\ -\sin t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\cos t \\ -\sin t + 1 \end{pmatrix}$

$\overrightarrow{CF} = \begin{pmatrix} -\sin t + 1 \\ \cos t \end{pmatrix}$

$\overrightarrow{OF} = \overrightarrow{OC} + \overrightarrow{CF} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + \begin{pmatrix} -\sin t + 1 \\ \cos t \end{pmatrix} = \begin{pmatrix} 1 - \sin t + \cos t \\ \sin t + \cos t \end{pmatrix}$