16.3 Optimaliseren

Opgave 34:

a. $f(x) = \dfrac{x-1}{x+2}$

$f'(x) = \dfrac{(x+2)\cdot 1 - (x-1)\cdot 1}{(x+2)^2} = \dfrac{x+2-x+1}{(x+2)^2} = \dfrac{3}{(x+2)^2}$

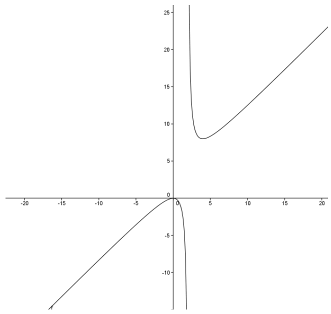
b. $(x+2)^2 > 0$ op het domein dus $\dfrac{3}{(x+2)^2} > 0$ op het domein

c. de grafiek van f blijft stijgen

Opgave 35:

a. $y = \dfrac{x^2}{x-2}$

$y' = \dfrac{(x-2)\cdot 2x - x^2 \cdot 1}{(x-2)^2} = \dfrac{2x^2 - 4x - x^2}{(x-2)^2} = \dfrac{x^2 - 4x}{(x-2)^2} = 0$



$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad \vee \quad x = 4$

max $f(0) = 0$

min $f(4) = 8$

b. $y_A = y(3) = 9$

$rc = y'(3) = -3$

$y = -3x + b$ door $(3,9)$

$9 = -9 + b$

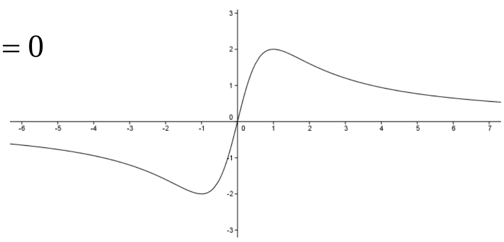
$18 = b$

$y = -3x + 18$

Opgave 36:

a. $y = \dfrac{4x}{x^2+1}$

$y' = \dfrac{(x^2+1)\cdot 4 - 4x \cdot 2x}{(x^2+1)^2} = \dfrac{4x^2 + 4 - 8x^2}{(x^2+1)^2} = \dfrac{4-4x^2}{(x^2+1)^2} = 0$



$4 - 4x^2 = 0$

$-4x^2 = -4$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

min $y(-1) = -2$

max $y(1) = 2$

b. $y = 2x + \dfrac{8}{x} = 2x + 8x^{-1}$

$y' = 2 - 8x^{-2} = 2 - \dfrac{8}{x^2} = 0$

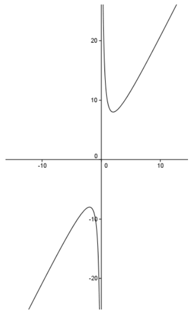
$-\dfrac{8}{x^2} = -2$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

max $y(-2) = -8$

min $y(2) = 8$



c. $y = 1{,}5x^2 + \dfrac{24}{x^2} = 1{,}5x^2 + 24x^{-2}$

$y' = 3x - 48x^{-3} = 3x - \dfrac{48}{x^3} = 0$

$-\dfrac{48}{x^3} = -3x$

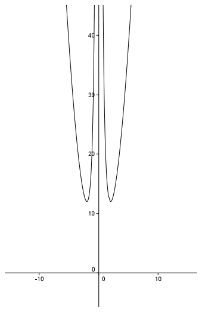
$-3x^4 = -48$

$x^4 = 16$

$x = 2 \quad \vee \quad x = -2$

min $y(-2) = 12$

min $y(2) = 12$



Opgave 37:

a. $C = \dfrac{0{,}16t}{t^2 + 4t + 4}$

$C' = \dfrac{(t^2 + 4t + 4) \cdot 0{,}16 - 0{,}16t \cdot (2t + 4)}{(t^2 + 4t + 4)^2}$

$= \dfrac{0{,}16t^2 + 0{,}64t + 0{,}64 - 0{,}32t^2 - 0{,}64t}{(t^2 + 4t + 4)^2}$

$= \dfrac{-0{,}16t^2 + 0{,}64}{(t^2 + 4t + 4)^2}$

$C'(0) = 0{,}04 > 0$ dus de concentratie neemt toe

b. $C' = \dfrac{-0{,}16t^2 + 0{,}64}{(t^2 + 4t + 4)^2} = 0$

$-0{,}16t^2 + 0{,}64 = 0$

$-0{,}16t^2 = -0{,}64$

$t^2 = 4$

$t = 2$

c. $C'(5) = -0{,}00140$

$C'(10) = -0{,}00074$

$\frac{-0{,}00074}{-0{,}00140} = 1{,}9$ dus inderdaad ongeveer 2 keer zo groot

d. $C(24) = 0{,}0057 > 0{,}005$ dus ja

Opgave 38:

a. $V(0) = 55$

b. $V = \dfrac{1000t + 3000}{t^2 + 16t + 64} + 8$

$V' = \dfrac{(t^2 + 16t + 64) \cdot 1000 - (1000t + 3000)(2t + 16)}{(t^2 + 16t + 64)^2}$

$= \dfrac{1000t^2 + 16000t + 64000 - (2000t^2 + 16000t + 6000t + 48000)}{(t^2 + 16t + 64)^2}$

$= \dfrac{1000t^2 + 16000t + 64000 - 2000t^2 - 22000t - 48000}{(t^2 + 16t + 64)^2}$

$= \dfrac{-1000t^2 - 6000t + 16000}{(t^2 + 16t + 64)^2}$

$V'(0) = 3{,}9 > 0$ dus V stijgt op $t = 0$

c. $V' = \dfrac{-1000t^2 - 6000t + 16000}{(t^2 + 16t + 64)^2} = 0$

$-1000t^2 - 6000t + 16000 = 0$

$t^2 + 6t - 16 = 0$

$(t + 8)(t - 2) = 0$

$t = -8 \quad \vee \quad t = 2$

dus op $t = 2$

d. vul een groot getal in voor t, dan geldt: $V \approx 8$

Opgave 39:

a. $T(4) - T(3) = 39{,}09 - 38{,}71 = 0{,}38°\text{ C}$

b. $T = 37 + \dfrac{45t}{t^2 + 70}$

$T' = \dfrac{(t^2 + 70) \cdot 45 - 45t \cdot 2t}{(t^2 + 70)^2} = \dfrac{45t^2 + 3150 - 90t^2}{(t^2 + 70)^2} = \dfrac{3150 - 45t^2}{(t^2 + 70)^2}$

2 mei 17.30 uur is $t = 29{,}5$

$T'(29{,}5) = -0{,}04$ dus de temperatuur daalt 0,04ºC

c. $T' = \dfrac{3150 - 45t^2}{(t^2 + 70)^2} = 0$

$3150 - 45t^2 = 0$

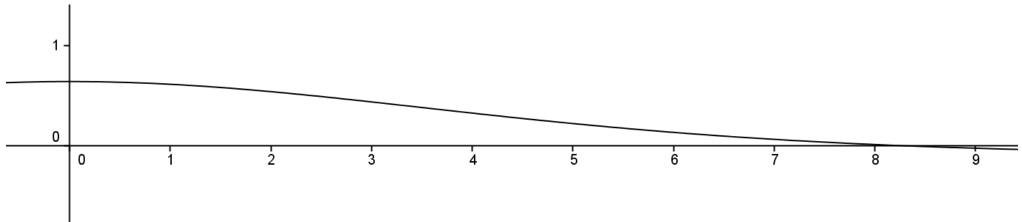
$-45t^2 = -3150$

$t^2 = 70$

$t = \sqrt{70}$

$T(\sqrt{70}) = 39{,}7°\text{ C}$

d.



op het eerste stuk is er sprake van een toenemende daling, vanaf ongeveer $t = 4{,}5$ is er sprake van een afnemende daling

Opgave 40:

a. $K_{bodem} = 24x^2$

b. $K_{zijkanten} = 4 \cdot x \cdot y \cdot 18 = 72xy$

c. $K_{totaal} = 24x^2 + 72xy = 288$

$72xy = 288 - 24x^2$

$3xy = 12 - x^2$

$y = \frac{12 - x^2}{3x}$

d. $I = x^2 y = x^2 \cdot \frac{12 - x^2}{3x} = \frac{12x^2 - x^4}{3x} = 4x - \frac{1}{3}x^3$

e. $I' = 4 - x^2 = 0$

$-x^2 = -4$

$x^2 = 4$

$x = 2$

$y = 1\frac{1}{3}$

dus 2 bij 2 bij $1\frac{1}{3}$ m

$I = 5\frac{1}{3}$

Opgave 41:

a. er kunnen zo gevaarlijke situaties ontstaan. Voor de veiligheid moeten de auto's meer afstand nemen als de snelheid groter is.

b. $t = \frac{12{,}5 + 4}{10} = 1{,}65$ sec

c. per auto $t = \frac{18 + 4}{12} = \frac{11}{6}$ sec

per uur: $\frac{3600}{\frac{11}{6}} = 1964$ auto's

d. per auto: $4 + r$ meter

per auto: $\frac{4 + r}{v}$ sec

per uur: $\frac{3600}{\frac{4+r}{v}} = \frac{3600v}{4 + r}$ auto's

e. $r = 0{,}125 \cdot 16^2 = 32$ meter

$Q = \frac{3600 \cdot 16^2}{4 + 32} = 1600$

f. $v = 54 \ ^{km}/_{uur} = 15 \ ^{m}/_{s}$

$r = 0{,}125 \cdot 15^2 = 28{,}125$

$Q = \frac{3600 \cdot 15}{32{,}125} = 1681$

g. $Q = \frac{3600v}{4 + 0{,}125v^2}$

$$Q' = \frac{(4+0{,}125v^2)\cdot 3600 - 3600v\cdot 0{,}25v}{(4+0{,}125v^2)^2}$$
$$= \frac{14400 + 450v^2 - 900v^2}{(4+0{,}125v^2)^2}$$
$$= \frac{14400 - 450v^2}{(4+0{,}125v^2)^2}$$

h. $\frac{14400 - 450v^2}{(4+0{,}125v^2)^2} = 0$

$14400 - 450v^2 = 0$

$-450v^2 = -14400$

$v^2 = 32$

$v = \sqrt{32} = 5{,}66\ {}^{m}/_{s} = 20{,}4\ {}^{km}/_{uur}$

$Q = 2546$

Opgave 42:

a. $W_{totaal} = 3v^2 + 7{,}5\cdot 10^{-4}\cdot\frac{(4\cdot 10^6)^2}{v^2} = 3v^2 + \frac{1{,}2\cdot 10^{10}}{v^2} = 3v^2 + 1{,}2\cdot 10^{10}\cdot v^{-2}$

$\frac{dW}{dv} = 6v - 2{,}4\cdot 10^{10}\cdot v^{-3} = 6v - \frac{2{,}4\cdot 10^{10}}{v^3} = \frac{6v^4 - 2{,}4\cdot 10^{10}}{v^3}$

b. $\frac{6v^4 - 2{,}4\cdot 10^{10}}{v^3} = 0$

$6v^4 - 2{,}4\cdot 10^{10} = 0$

$6v^4 = 2{,}4\cdot 10^{10}$

$v^4 = 0{,}4\cdot 10^{10}$

$v = 251{,}5\ {}^{m}/_{s} = 905\ {}^{km}/_{uur}$

c. $3v^2 = \frac{1{,}2\cdot 10^{10}}{v^2}$

$3v^4 = 1{,}2\cdot 10^{10}$

$v^4 = 0{,}4\cdot 10^{10}$

$v = 251{,}5\ {}^{m}/_{s} = 905\ {}^{km}/_{uur}$

Opgave 43:

a. boven zee heeft een vogel te maken met een neerwaartse luchtstroom en het kost een vogel dan ook meer energie om over zee te vliegen.

b. $EB = \sqrt{5^2 + 12^2} = 13$

dus $13\cdot 50 = 650$ kJ

Opgave 44:

a. $EC = \sqrt{5^2 + x^2} = \sqrt{25 + x^2}$

$BC = 12 - x$

$Energie = 50\sqrt{25 + x^2} + 32(12 - x)$

b. $50\sqrt{25+x^2}+32(12-x)=640$

$y_1=50\sqrt{25+x^2}+32(12-x)$ en $y_2=640$

intersect geeft $x=11{,}28$

c. de optie minimum geeft: $x=4{,}16$

$E=576$

Opgave 45:

a. $y=6\sqrt{x^2+288}+10-2x$

$\frac{dy}{dx}=6\cdot\frac{1}{2\sqrt{x^2+288}}\cdot 2x-2=\frac{6x}{\sqrt{x^2+288}}-2$

b. $\frac{6x}{\sqrt{x^2+288}}-2=0$

$\frac{6x}{\sqrt{x^2+288}}=2$

$6x=2\sqrt{x^2+288}$

$3x=\sqrt{x^2+288}$

$9x^2=x^2+288$

$8x^2=288$

$x^2=36$

$x=6$

$y=106$

Opgave 46:

a. $BE=\sqrt{5^2+10^2}=\sqrt{125}=11{,}180\text{ km}=11180\text{ m}$

$K=11180\cdot 140=1565200$

b. $K=10000\cdot 100+5000\cdot 140=1700000$

c. $CE=\sqrt{5^2+2^2}=\sqrt{29}=5{,}385\text{ km}=5385\text{ m}$

$K=5385\cdot 140+8000\cdot 100=1553900$

d. $BP=10000-1000x$

$K=(10000-1000x)\cdot 100=1000000-100000x$

e. $PE=\sqrt{5^2+x^2}=\sqrt{25+x^2}$

$K=140\cdot 1000\sqrt{25+x^2}=140000\sqrt{25+x^2}$

f. $TK=140000\sqrt{25+x^2}+1000000-100000x$

g. $TK'=140000\cdot\frac{1}{2\sqrt{25+x^2}}\cdot 2x-100000=\frac{140000x}{\sqrt{25+x^2}}-100000=0$

$y_1=\frac{140000x}{\sqrt{25+x^2}}-100000$ de optie zero geeft $x=5{,}103$

Opgave 47:

a. $PE=\sqrt{x^2+200^2}=\sqrt{x^2+40000}$

$PF=2000-x$

$K = 65\sqrt{x^2 + 40000} + 50(2000 - x) = 65\sqrt{x^2 + 40000} + 100000 - 50x$

b. $K' = 65 \cdot \dfrac{1}{2\sqrt{x^2 + 400000}} \cdot 2x - 50 = \dfrac{65x}{\sqrt{x^2 + 40000}} - 50 = 0$

$y_1 = \dfrac{65}{\sqrt{x^2 + 40000}} - 50$ de optie zero geeft $x = 241$

Opgave 48:

a. $p(q) = 1560 - a\sqrt{q}$

$R = p \cdot q = (1560 - a\sqrt{q}) \cdot q = 1560q - aq\sqrt{q} = 1560q - aq^{1\frac{1}{2}}$

$R' = 1560 - 1\tfrac{1}{2}aq^{\frac{1}{2}} = 1560 - 1\tfrac{1}{2}a\sqrt{q}$

$R'(169) = 1560 - 1\tfrac{1}{2}a\sqrt{169} = 1560 - 19{,}5a = 0$

$-19{,}5a = -1560$

$a = 80$

b. $p = 1560 - 92\sqrt{q} = 548$

$-92\sqrt{q} = -1012$

$\sqrt{q} = 11$

$q = 121$

$W = R - K = 1560q - 92q\sqrt{q} - (250 + bq) = 1560q - 92q\sqrt{q} - 250 - bq$

$W' = 1560 - 138q^{\frac{1}{2}} - b = 1560 - 138\sqrt{q} - b$

$W'(121) = 1560 - 138\sqrt{121} - b = 0$

$-b = -1560 + 138\sqrt{121} = -42$

$b = 42$