

16.2 Regels voor het differentiëren

Opgave 17:

- a. $p'(x) = 6x^2 + 2x$
b. $f'(x) = 2x$
 $g'(x) = 2$
 $f'(x) \cdot g'(x) = 2x \cdot 2 = 4x$
c. nee

Opgave 18:

- a. $f(x) = x(2x+1)^3$
 $f'(x) = 1 \cdot (2x+1)^3 + x \cdot 3(2x+1)^2 \cdot 2 = (2x+1)^3 + 6x(2x+1)^2$
b. $g(x) = x\sqrt{1-3x}$
 $g'(x) = 1 \cdot \sqrt{1-3x} + x \cdot \frac{1}{2\sqrt{1-3x}} \cdot -3 = \sqrt{1-3x} - \frac{3x}{2\sqrt{1-3x}}$
c. $h(x) = 5x + x^2(x^2+1)^{1,8}$
 $h'(x) = 5 + 2x \cdot (x^2+1)^{1,8} + x^2 \cdot 1,8(x^2+1)^{0,8} \cdot 2x = 5 + 2x(x^2+1)^{1,8} + 3,6x^3(x^2+1)^{0,8}$
d. $k(x) = 6(2x-1)\sqrt{2x-1} = 6(2x-1)^{\frac{3}{2}}$
 $k'(x) = 9(2x-1)^{\frac{1}{2}} \cdot 2 = 18\sqrt{2x-1}$

Opgave 19:

- a. $y = (x+3)(2x-5)^2$
 $y' = 1 \cdot (2x-5)^2 + (x+3) \cdot 2(2x-5) \cdot 2 = (2x-5)^2 + 4(x+3)(2x-5)$
b. $y = (x^2-1)\sqrt{x^2+4}$
 $y' = 2x \cdot \sqrt{x^2+4} + (x^2-1) \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x = 2x\sqrt{x^2+4} + \frac{x(x^2-1)}{\sqrt{x^2+4}}$
c. $K = 7q\sqrt{100-q}$
 $K' = 7 \cdot \sqrt{100-q} + 7q \cdot \frac{1}{2\sqrt{100-q}} \cdot -1 = 7\sqrt{100-q} - \frac{7q}{2\sqrt{100-q}}$
d. $A = 5t^3 - t\sqrt{2-3t}$
 $A' = 15t^2 - 1 \cdot \sqrt{2-3t} - t \cdot \frac{1}{2\sqrt{2-3t}} \cdot -3 = 15t^2 - \sqrt{2-3t} + \frac{3t}{2\sqrt{2-3t}}$

Opgave 20:

- a. $[5x^3]' = [5 \cdot x^3]' = 0 \cdot x^3 + 5 \cdot [x^3]' = 5 \cdot [x^3]'$
b. $[a \cdot f(x)]' = [a]' \cdot f(x) + a \cdot [f(x)]' = 0 \cdot f(x) + a \cdot [f(x)]' = a \cdot [f(x)]'$

Opgave 21:

- a. $8(x+1)^2 + 3(x+1) = (x+1)(8(x+1) + 3) = (x+1)(8x+8+3) = (x+1)(8x+11)$
b. $6(2x-1)^5 - 2(2x-1)^4 = (2x-1)^4(6(2x-1) - 2) = (2x-1)^4(12x-6-2) = (2x-1)^4(12x-8)$

c. $(x+3)\sqrt{5-x} - 2\sqrt{5-x} = \sqrt{5-x}(x+3-2) = (x+1)\sqrt{5-x}$

d. $(2x-3)(x-7)^5 - (x-7)^3 = (x-7)^3((2x-3)(x-7)^2 - 1)$
 $= (x-7)^3((2x-3)(x^2 - 14x + 49) - 1)$
 $= (x-7)^3(2x^3 - 28x^2 + 98x - 3x^2 + 42x - 147 - 1)$
 $= (x-7)^3(2x^3 - 31x^2 + 140x - 148)$

Opgave 22:

a. $\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{3}{\sqrt{x}} = \frac{x+3}{\sqrt{x}}$

b. $\sqrt{1-x} + \frac{3}{\sqrt{1-x}} = \frac{1-x}{\sqrt{1-x}} + \frac{3}{\sqrt{1-x}} = \frac{4-x}{\sqrt{1-x}}$

c. $\sqrt{x^2+3} + \frac{5x}{\sqrt{x^2+3}} = \frac{x^2+3}{\sqrt{x^2+3}} + \frac{5x}{\sqrt{x^2+3}} = \frac{x^2+5x+3}{\sqrt{x^2+3}}$

d. $2\sqrt{2x-1} - \frac{4}{\sqrt{2x-1}} = \frac{2(2x-1)}{\sqrt{2x-1}} - \frac{4}{\sqrt{2x-1}} = \frac{4x-2-4}{\sqrt{2x-1}} = \frac{4x-6}{\sqrt{2x-1}}$

Opgave 23:

a. $f(x) = 4x(3x-1)^5$
 $f'(x) = 4 \cdot (3x-1)^5 + 4x \cdot 5(3x-1)^4 \cdot 3 = 4(3x-1)^5 + 60x(3x-1)^4$
 $= (3x-1)^4(4(3x-1) + 60x) = (3x-1)^4(12x - 4 + 60x) = (3x-1)^4(72x - 4)$

b. $g(x) = x(1-x^2)^4$
 $g'(x) = 1 \cdot (1-x^2)^4 + x \cdot 4(1-x^2)^3 \cdot -2x = (1-x^2)^3(1-x^2 - 8x^2) = (1-x^2)^3(1-9x^2)$

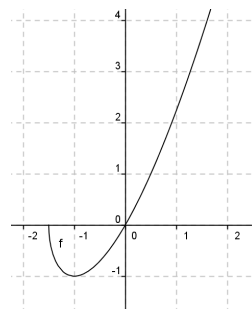
c. $h(x) = x\sqrt{2-2x}$
 $h'(x) = 1 \cdot \sqrt{2-2x} + x \cdot \frac{1}{2\sqrt{2-2x}} \cdot -2 = \sqrt{2-2x} - \frac{x}{\sqrt{2-2x}} = \frac{2-2x}{\sqrt{2-2x}} - \frac{x}{\sqrt{2-2x}}$
 $= \frac{2-3x}{\sqrt{2-2x}}$

Opgave 24:

a. $f(x) = x\sqrt{3+2x}$
 $f'(x) = 1 \cdot \sqrt{3+2x} + x \cdot \frac{1}{2\sqrt{3+2x}} \cdot 2 = \sqrt{3+2x} + \frac{x}{\sqrt{3+2x}} = \frac{3+2x}{\sqrt{3+2x}} + \frac{x}{\sqrt{3+2x}}$
 $= \frac{3+3x}{\sqrt{3+2x}}$

b. $\frac{3+3x}{\sqrt{3+2x}} = 0$
 $3+3x = 0$
 $3x = -3$
 $x = -1$
 $\min f(-1) = -1$

c. $y_A = 9$
 $f'(3) = 4$



$$y = 4x + b \text{ door } (3,9)$$

$$9 = 12 + b$$

$$b = -3$$

$$k: y = 4x - 3$$

Opgave 25:

a. $g(x) = x(2-x)^4 + 2$

$$g'(x) = 1 \cdot (2-x)^4 + x \cdot 4(2-x)^3 \cdot -1$$

$$= (2-x)^4 - 4x(2-x)^3$$

$$= (2-x)^3(2-x-4x)$$

$$= (2-x)^3(2-5x)$$

b. $g'(x) = (2-x)^3(2-5x) = 0$

$$2-x=0 \quad \vee \quad 2-5x=0$$

$$-x=-2 \quad \vee \quad -5x=-2$$

$$x=2 \quad \vee \quad x=\frac{2}{5}$$

$$\max g\left(\frac{2}{5}\right) = 4,62144$$

$$\min g(2) = 2$$

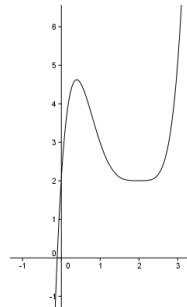
c. $P(0,2)$

$$g'(0) = 16$$

$$y = 16x + b \text{ door } (0,2)$$

$$2 = b$$

$$l: y = 16x + 2$$



Opgave 26:

a. $t'(x) = 3$

$$n'(x) = 1$$

$$\frac{t'(x)}{n'(x)} = \frac{3}{1} = 3$$

- b. als $\frac{t'(x)}{n'(x)} = 3$ dan is de helling van de grafiek constant, dus zou de grafiek een rechte lijn zijn, wat niet het geval is.

Opgave 27:

a. $y = \frac{x+1}{x-4}$

$$y' = \frac{(x-4) \cdot 1 - (x+1) \cdot 1}{(x-4)^2} = \frac{x-4-x-1}{(x-4)^2} = \frac{-5}{(x-4)^2}$$

b. $y = 7x + \frac{-2x+3}{x+7}$

$$y' = 7 + \frac{(x+7) \cdot -2 - (-2x+3) \cdot 1}{(x+7)^2} = 7 + \frac{-2x-14+2x-3}{(x+7)^2} = 7 - \frac{17}{(x+7)^2}$$

c. $y = \frac{3x^2}{6-x} + 4x^3$

$$y' = \frac{(6-x) \cdot 6x - 3x^2 \cdot -1}{(6-x)^2} + 12x^2 = \frac{36x - 6x^2 + 3x^2}{(6-x)^2} + 12x^2 = \frac{36x - 3x^2}{(6-x)^2} + 12x^2$$

d. $y = \frac{4x + x^3}{8x^2 - 5}$

$$y' = \frac{(8x^2 - 5) \cdot (4 + 3x^2) - (4x + x^3) \cdot 16x}{(8x^2 - 5)^2}$$

$$= \frac{32x^2 + 24x^4 - 20 - 15x^2 - 64x^2 - 16x^4}{(8x^2 - 5)^2}$$

$$= \frac{8x^4 - 47x^2 - 20}{(8x^2 - 5)^2}$$

Opgave 28:

a. $A = \frac{-2}{3+2t} + 5t$

$$A' = \frac{(3+2t) \cdot 0 - -2 \cdot 2}{(3+2t)^2} + 5 = \frac{4}{(3+2t)^2} + 5$$

b. $K = 2q - \frac{1}{q+8}$

$$K' = 2 - \frac{(q+8) \cdot 0 - 1 \cdot 1}{(q+8)^2} = 2 + \frac{1}{(q+8)^2}$$

c. $P = \left(\frac{2}{1+q}\right)^3 = u^3$ met $u = \frac{2}{1+q}$ dus $u' = \frac{(1+q) \cdot 0 - 2 \cdot 1}{(1+q)^2} = \frac{-2}{(1+q)^2}$

$$P' = 3u^2 \cdot u' = 3 \cdot \left(\frac{2}{1+q}\right)^2 \cdot \frac{-2}{(1+q)^2} = 3 \cdot \frac{4}{(1+q)^2} \cdot \frac{-2}{(1+q)^2} = \frac{-24}{(1+q)^4}$$

d. $N = \left(\frac{2t}{t-1}\right)^4 = u^4$ met $u = \frac{2t}{t-1}$ dus $u' = \frac{(t-1) \cdot 2 - 2t \cdot 1}{(t-1)^2} = \frac{2t - 2 - 2t}{(t-1)^2} = \frac{-2}{(t-1)^2}$

$$N' = 4u^3 \cdot u' = 4 \cdot \left(\frac{2t}{t-1}\right)^3 \cdot \frac{-2}{(t-1)^2} = 4 \cdot \frac{8t^3}{(t-1)^3} \cdot \frac{-2}{(t-1)^2} = \frac{-64t^3}{(t-1)^5}$$

Opgave 29:

a. $P(4,5) = 140,9$

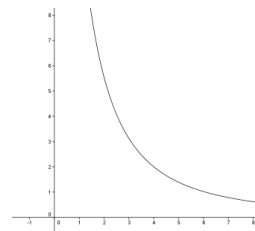
$$P(6,5) = 143,3$$

$$\frac{143,3 - 140,9}{140,9} \cdot 100\% = 1,7\%$$

b. $P' = -\frac{(1+x) \cdot 0 - 50 \cdot 1}{(1+x)^2} = \frac{50}{(1+x)^2}$

de grafiek van P' ligt boven de x -as dus de grafiek van P is stijgend

de grafiek van P' daalt, dus er is sprake van een afnemende stijging bij de grafiek van P



$$c. \quad P' = \frac{50}{(1+x)^2} < 0,8$$

$$\text{neem } y_1 = \frac{50}{(1+x)^2} \text{ en } y_2 = 0,8$$

intersect geeft $x = 6,9$, dus voor $x > 6,9$

Opgave 30:

$$a. \quad \frac{20p + 1600}{4p + 5} = 20$$

$$20(4p + 5) = 20p + 1600$$

$$80p + 100 = 20p + 1600$$

$$60p = 1500$$

$$p = 25 \text{ dus } 25 \text{ euro}$$

$$b. \quad q' = \frac{(4p + 5) \cdot 20 - (20p + 1600) \cdot 4}{(4p + 5)^2} = \frac{80p + 100 - 80p - 6400}{(4p + 5)^2} = \frac{-6300}{(4p + 5)^2}$$

voor iedere waarde van p geldt $q' < 0$ dus de grafiek van q daalt
dus bij een verhoging van de prijs zal de verkoop afnemen

$$c. \quad q'(18) = -1,06 \text{ dus met } 1,06 \text{ euro per kist}$$

Opgave 31:

$$a. \quad \left[\frac{dP}{dt} \right]_{t=4} = 5,19$$

$$\frac{5,19}{7} = 0,7\% \text{ per dag}$$

$$b. \quad P' = \frac{(t^2 + 1) \cdot (200t - 100) - 100(t^2 - t + 1) \cdot 2t}{(t^2 + 1)^2}$$

$$= \frac{200t^3 - 100t^2 + 200t - 100 - 200t^3 + 200t^2 - 200t}{(t^2 + 1)^2}$$

$$= \frac{100t^2 - 100}{(t^2 + 1)^2}$$

$P'(1) = 0$ in de grafiek zie je dat je te maken hebt met een minimum

$$c. \quad y_1 = \frac{100(x^2 - x + 1)}{x^2 + 1} \text{ en } y_2 = 98$$

intersect geeft 49,98 dus $49,98 \cdot 7 = 350$ dagen

$$d. \quad \left[\frac{dP}{dt} \right]_{t=8} = 1,49$$

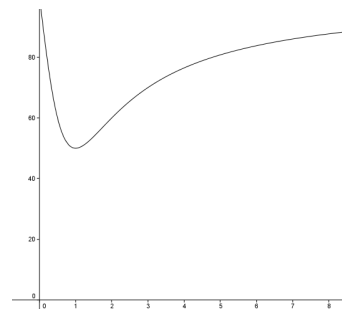
$$P(8) = 87,69$$

$$87,69 + 1,49t = 100$$

$$1,49t = 12,31$$

$$t = 8,26 \text{ dus } 8,26 \cdot 7 = 58$$

dus na $8 \cdot 7 + 58 = 114$ dagen



Opgave 32:

a. $y = 4(3x - 2)^3 = 4u^3$ met $u = 3x - 2$ dus $u' = 3$
 $y' = 12u^2 \cdot u' = 12(3x - 2)^2 \cdot 3 = 36(3x - 2)^2$

b. $y = \frac{x+8}{x^2} = \frac{x}{x^2} + \frac{8}{x^2} = x^{-1} + 8x^{-2}$
 $y' = -x^{-2} - 16x^{-3} = \frac{-1}{x^2} - \frac{16}{x^3}$

c. $y = 4\sqrt{x^2 - 3} = 4\sqrt{u} = 4u^{\frac{1}{2}}$ met $u = x^2 - 3$ dus $u' = 2x$
 $y' = 2u^{-\frac{1}{2}} \cdot u' = \frac{2}{\sqrt{u}} \cdot u' = \frac{2}{\sqrt{x^2 - 3}} \cdot 2x = \frac{4x}{\sqrt{x^2 - 3}}$

d. $y = 6x\sqrt{5-4x}$
 $y' = 6 \cdot \sqrt{5-4x} + 6x \cdot \frac{1}{2\sqrt{5-4x}} \cdot -4 = 6\sqrt{5-4x} - \frac{12}{\sqrt{5-4x}}$

e. $y = \frac{3}{(2x-7)^4} = 3(2x-7)^{-4} = 3u^{-4}$ met $u = 2x-7$ dus $u' = 2$
 $y' = -12u^{-5} \cdot u' = \frac{-12}{u^5} \cdot u' = \frac{-12}{(2x-7)^5} \cdot 2 = \frac{-24}{(2x-7)^5}$

f. $y = 4x^2 \cdot \sqrt{x} = 4x^2 \cdot x^{\frac{1}{2}} = 4x^{\frac{5}{2}}$
 $y' = 10x^{\frac{3}{2}} = 10x\sqrt{x}$

g. $y = 5x + \sqrt{1-x}$
 $y' = 5 + \frac{1}{2\sqrt{1-x}} \cdot -1 = 5 - \frac{1}{2\sqrt{1-x}}$

h. $y = \frac{3}{x^2} + \frac{2}{3x} = 3x^{-2} + \frac{2}{3}x^{-1}$
 $y' = -6x^{-3} - \frac{2}{3}x^{-2} = \frac{-6}{x^3} - \frac{2}{3x^2}$

i. $y = 5x\sqrt{5x} = (5x)^{\frac{3}{2}} = u^{\frac{3}{2}}$ met $u = 5x$ dus $u' = 5$
 $y' = 1\frac{1}{2}u^{\frac{1}{2}} \cdot u' = 1\frac{1}{2}\sqrt{5x} \cdot 5 = 7\frac{1}{2}\sqrt{5x}$

Opgave 33:

a. $f(x) = x^2(1-x)^6$
 $f'(x) = 2x \cdot (1-x)^6 + x^2 \cdot 6(1-x)^5 \cdot -1 = 2x(1-x)^6 - 6x^2(1-x)^5$
 $= (1-x)^5(2x(1-x) - 6x^2) = (1-x)^5(2x - 2x^2 - 6x^2) = (1-x)^5(2x - 8x^2)$

b. $g(x) = 2x^2\sqrt{1-x^2}$
 $g'(x) = 4x \cdot \sqrt{1-x^2} + 2x^2 \cdot \frac{1}{2\sqrt{1-x^2}} \cdot -2x = 4x\sqrt{1-x^2} - \frac{2x^3}{\sqrt{1-x^2}}$
 $= \frac{4x(1-x^2)}{\sqrt{1-x^2}} - \frac{2x^3}{\sqrt{1-x^2}} = \frac{4x - 4x^3 - 2x^3}{\sqrt{1-x^2}} = \frac{4x - 6x^3}{\sqrt{1-x^2}}$

c. $h(x) = \frac{x}{(2x-1)^3}$

$$\begin{aligned}h'(x) &= \frac{(2x-1)^3 \cdot 1 - x \cdot 3(2x-1)^2 \cdot 2}{(2x-1)^6} \\&= \frac{(2x-1)^3 - 6x(2x-1)^2}{(2x-1)^6} \\&= \frac{(2x-1)^2(2x-1-6x)}{(2x-1)^6} \\&= \frac{-4x-1}{(2x-1)^4}\end{aligned}$$