14.4 Exponentiële en logaritmische formules.

Opgave 55:

a. $\frac{\Delta N}{\Delta t} = \frac{90-50}{12-8} = 10$

b. $g^4 = \frac{90}{50} = 1{,}8$

$g = \sqrt[4]{1{,}8} = 1{,}16$

Opgave 56:

a. $\frac{\Delta N_1}{\Delta t} = \frac{942-750}{20-12} = 24$

$N_1 = 24t + b$ door $(12{,}750)$

$750 = 288 + b$

$b = 462$

$N_1 = 24t + 462$

$g^8 = \frac{942}{750} = 1{,}256$

$g = \sqrt[8]{1{,}256} = 1{,}029$

$b = \frac{750}{1{,}029^{12}} = 533$

$N_2 = 533 \cdot 1{,}029^t$

b. $N_2 = 2 \cdot N_1$

neem $y_1 = 2 \cdot (24x + 462)$ en $y_2 = 533 \cdot 1{,}029^x$

intersect geeft: $x = 74{,}7$ dus voor $t = 74{,}7$

Opgave 57:

a. $\frac{\Delta N_1}{\Delta t} = \frac{1820-2180}{10-6} = -90$

$N_1 = -90t + b$ door $(6{,}2180)$

$2180 = -540 + b$

$b = 2720$

$N_1 = -90t + 2720$

$g^4 = \frac{1820}{2180} = 0{,}835$

$g = \sqrt[4]{0{,}835} = 0{,}956$

$b = \frac{2180}{0{,}956^6} = 2858$

$N_2 = 2858 \cdot 0{,}956^t$

b. $N_2 = 2 \cdot N_1$

neem $y_1 = 2 \cdot (-90x + 2720)$ en $y_2 = 2858 \cdot 0{,}956^x$

intersect geeft: $x = 25{,}1$ dus voor $t = 25{,}1$

Opgave 58:

a. $\frac{1}{x^3} = x^{-3}$

b. $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

c. $\sqrt{x} = x^{\frac{1}{2}}$

d. $x^3 \cdot x^5 = x^8$

e. $x^6 : x^2 = x^4$

f. $x^3 \cdot \sqrt{x} = x^3 \cdot x^{\frac{1}{2}} = x^{3\frac{1}{2}}$

Opgave 59:

a. $y = 5x^4 \cdot \sqrt{x} = 5x^4 \cdot x^{\frac{1}{2}} = 5x^{4\frac{1}{2}}$

b. $y = \frac{5}{x} \cdot x^{1,3} = 5x^{-1} \cdot x^{1,3} = 5x^{0,3}$

c. $y = (3x^{-1,8})^4 \cdot 2x^{3,6} = 3^4 \cdot x^{-7,2} \cdot 2x^{3,6} = 162x^{-3,6}$

Opgave 60:

a. $N = 25 \cdot 1{,}4^{3t-2} = 25 \cdot 1{,}4^{3t} \cdot 1{,}4^{-2} = 25 \cdot (1{,}4^3)^t \cdot 0{,}51 = 12{,}76 \cdot 2{,}74^t$

b. $N = 180 \cdot 0{,}8^{5-t} = 180 \cdot 0{,}8^5 \cdot 0{,}8^{-t} = 58{,}98 \cdot (0{,}8^{-1})^t = 58{,}98 \cdot 1{,}25^t$

c. $N = 92{,}6 \cdot 1{,}7^{-1,2t+0,5} = 92{,}6 \cdot 1{,}7^{-1,2t} \cdot 1{,}7^{0,5} = 120{,}74 \cdot (1{,}7^{-1,2})^t = 120{,}74 \cdot 0{,}53^t$

Opgave 61:

a. $y = 18 \cdot (2x^2)^{0,3} \cdot (5z)^{0,6}$
$= 18 \cdot 2^{0,3} \cdot x^{0,6} \cdot 5^{0,6} \cdot z^{0,6}$
$= 58{,}21x^{0,6}z^{0,6}$

b. $N = 18 - 5(6 - 1{,}5^{4t})$
$= 18 - 30 + 5 \cdot 1{,}5^{4t}$
$= -12 + 5 \cdot (1{,}5^4)^t$
$= -12 + 5 \cdot 5{,}06^t$

c. $T = 27 \cdot 0{,}4^t (3 - 0{,}4^{2t})$
$= 27 \cdot 0{,}4^t \cdot 3 - 27 \cdot 0{,}4^t \cdot 0{,}4^{2t}$
$= 81 \cdot 0{,}4^t - 27 \cdot 0{,}4^{3t}$
$= 81 \cdot 0{,}4^t - 27 \cdot (0{,}4^3)^t$
$= 81 \cdot 0{,}4^t - 27 \cdot 0{,}64^t$

Opgave 62:

a. $L = 0{,}006 \cdot (5t)^{0,35} \cdot (5s)^{0,18}$
$= 0{,}006 \cdot 5^{0,35} \cdot t^{0,35} \cdot (\frac{5}{3}t^3)^{0,18}$
$= 0{,}006 \cdot 5^{0,35} \cdot t^{0,35} \cdot (\frac{5}{3})^{0,18} \cdot t^{0,54}$
$= 0{,}01t^{0,89}$

b. $y = 0{,}12 \cdot (3t)^{1,6} \cdot (\frac{1}{5}z)^{2,3}$
$= 0{,}12 \cdot (1{,}5z^{-0,4})^{1,6} \cdot (\frac{1}{5})^{2,3} \cdot z^{2,3}$

$= 0{,}12 \cdot 1{,}5^{1,6} \cdot z^{-0,64} \cdot (\frac{1}{5})^{2,3} \cdot z^{2,3}$

$= 0{,}01 \cdot z^{1,66}$

c. $y = \dfrac{500}{x^4}$

$= \dfrac{500}{(4\sqrt{20 + a^2})^4}$

$= \dfrac{500}{4^4 \cdot (20 + a^2)^2}$

$= 1{,}95(20 + a^2)^{-2}$

Opgave 63:

a. $x^5 = 18$

$x = \sqrt[5]{18}$

b. $\sqrt[3]{x} = 4$

$x = 4^3 = 64$

Opgave 64:

a. $5x^{-1,2} = 20$

$x^{-1,2} = 4$

$x = \sqrt[-1,2]{4} = 0{,}31$

b. $3 + 0{,}18x^{1,7} = 25$

$0{,}18x^{1,7} = 22$

$x^{1,7} = 122\frac{2}{9}$

$x = \sqrt[1,7]{122\frac{2}{9}} = 16{,}89$

c. $5 \cdot \sqrt[4]{x} - 7 = 43$

$5 \cdot \sqrt[4]{x} = 50$

$\sqrt[4]{x} = 10$

$x = 10^4 = 10000$

d. $3x^6 = 57$

$x^6 = 19$

$x = \sqrt[6]{19} = 1{,}63 \quad \vee \quad x = -1{,}63$

Opgave 65:

a. $y = 3x^{2,6}$

$\frac{1}{3}y = x^{2,6}$

$x = (\frac{1}{3}y)^{\frac{1}{2,6}} = 0{,}66y^{0,38}$

b. $y = 0{,}18 \cdot (3x)^{-1,4} = 0{,}18 \cdot 3^{-1,4} \cdot x^{-1,4} = 0{,}039 \cdot x^{-1,4}$

$x^{-1,4} = 25{,}86y$

$x = (25{,}86y)^{\frac{1}{-1,4}} = 25{,}86^{-0,71} \cdot y^{-0,71} = 0{,}10 \cdot y^{-0,71}$

c. $y = 7 \cdot \sqrt[5]{3x} = 7 \cdot (3x)^{\frac{1}{5}} = 7 \cdot 3^{\frac{1}{5}} \cdot x^{\frac{1}{5}} = 8{,}72x^{\frac{1}{5}}$

$x^{\frac{1}{5}} = 0{,}115y$

$x = (0{,}115y)^5 = 0{,}115^5 \cdot y^5 = 0{,}00002y^5$

d. $y = 1{,}9 \cdot (2x)^{3{,}6} \cdot (3x)^{-1{,}7} = 1{,}9 \cdot 2^{3{,}6} \cdot x^{3{,}6} \cdot 3^{-1{,}7} \cdot x^{-1{,}7} = 3{,}56 \cdot x^{1{,}9}$

$x^{1{,}9} = 0{,}28y$

$x = (0{,}28y)^{\frac{1}{1{,}9}} = 0{,}28^{0{,}53} \cdot y^{0{,}53} = 0{,}51y^{0{,}53}$

Opgave 66:

a. $P = 2{,}5q^{3{,}6}$

$q^{3{,}6} = 0{,}4P$

$q = (0{,}4P)^{\frac{1}{3{,}6}} = 0{,}4^{0{,}28} \cdot P^{0{,}28} = 0{,}78P^{0{,}28}$

b. $L = \frac{1}{6} \cdot \sqrt[3]{A} - 7$

$L + 7 = \frac{1}{6} \cdot \sqrt[3]{A}$

$6L + 42 = \sqrt[3]{A}$

$A = (6L + 42)^3$

c. $O = 8a^2$

$a^2 = 0{,}125O$

$a = \sqrt{0{,}125O} = 0{,}35O^{0{,}5}$

$K = 4 \cdot (0{,}35O^{0{,}5})^3 = 4 \cdot 0{,}35^3 \cdot O^{1{,}5} = 0{,}18O^{1{,}5}$

$O^{1{,}5} = 5{,}66K$

$O = (5{,}66K)^{\frac{1}{1{,}5}} = 5{,}66^{\frac{2}{3}} \cdot K^{\frac{2}{3}} = 3{,}17K^{0{,}67}$

Opgave 67:

a. ${}^3\log 81 = {}^3\log 3^4 = 4$

b. ${}^5\log 5 = {}^5\log 5^1 = 1$

c. ${}^2\log \frac{1}{2} = {}^2\log 2^{-1} = -1$

d. ${}^{10}\log 1000 = {}^{10}\log 10^3 = 3$

e. ${}^6\log 1 = {}^6\log 6^0 = 0$

f. ${}^3\log 3^{1{,}9} = 1{,}9$

Opgave 68:

a. $\log N = 1{,}17 + 0{,}3t$

$N = 10^{1{,}17+0{,}3t}$

$N = 10^{1{,}17} \cdot 10^{0{,}3t}$

$N = 15 \cdot (10^{0{,}3})^t$

$N = 15 \cdot 2{,}00^t$

b. $\log a = 2{,}16 + 1{,}3\log b$

$a = 10^{2{,}16+1{,}3\log b}$

$a = 10^{2{,}16} \cdot 10^{1{,}3\log b}$

$a = 145 \cdot 10^{\log b^{1{,}3}}$

$a = 145 \cdot b^{1{,}3}$

c. $^2\log p = 1{,}18 - 0{,}8 \cdot {}^2\log q$

$p = 2^{1{,}18 - 0{,}8 \cdot {}^2\log q}$

$p = 2^{1{,}18} \cdot 2^{-0{,}8 \cdot {}^2\log q}$

$p = 2{,}27 \cdot 2^{{}^2\log q^{-0{,}8}}$

$p = 2{,}27 \cdot q^{-0{,}8}$

d. $N = \log(3L + 6)$

$3L + 6 = 10^N$

$3L = -6 + 10^N$

$L = -2 + \frac{1}{3} \cdot 10^N$

Opgave 69:

a. $0{,}5k = \log(2T + 5) - 1{,}8$

$0{,}5k + 1{,}8 = \log(2T + 5)$

$2T + 5 = 10^{0{,}5k + 1{,}8}$

$2T + 5 = 10^{1{,}8} \cdot 10^{0{,}5k}$

$2T + 5 = 63{,}10 \cdot (10^{0{,}5})^k$

$2T + 5 = 63{,}10 \cdot 3{,}16^k$

$2T = -5 + 63{,}10 \cdot 3{,}16^k$

$T = -2{,}5 + 31{,}55 \cdot 3{,}16^k$

b. $3\log M = 1{,}6 + \log(4S - 1)$

$-1{,}6 + \log M^3 = \log(4S - 1)$

$4S - 1 = 10^{-1{,}6 + \log M^3}$

$4S - 1 = 10^{-1{,}6} \cdot 10^{\log M^3}$

$4S - 1 = 0{,}025 \cdot M^3$

$4S = 1 + 0{,}025M^3$

$S = 0{,}25 + 0{,}01M^3$

Opgave 70:

a. $S = 290\log(185 + 100) - 550 = 162$ cm

b. $290\log(g + 100) - 550 = 210$

$290\log(g + 100) = 760$

$\log(g + 100) = 2{,}62$

$g + 100 = 10^{2{,}62} = 418$

$g = 318$ cm

c. $S = 290\log(g + 100) - 550$

$S + 550 = 290\log(g + 100)$

$0{,}0034S + 1{,}90 = \log(g + 100)$

$g + 100 = 10^{0{,}0034S + 1{,}90}$

$g + 100 = 10^{1{,}90} \cdot 10^{0{,}0034S}$

$g + 100 = 78{,}8 \cdot (10^{0{,}0034})^S$

$g = -100 + 78{,}8 \cdot 1{,}008^S$

Opgave 71:

a. $\log N = 5{,}3 - 1{,}7\log 50 = 2{,}41$

$N = 10^{2{,}41} = 258$

b. $N = \frac{2000}{8} = 250$

$\log 250 = 5{,}3 - 1{,}7\log D$

$1{,}7\log D = 5{,}3 - \log 250 = 2{,}90$

$\log D = 1{,}71$

$D = 10^{1{,}71} = 51$ cm

c. $\log N = 5{,}3 - 1{,}7\log D$

$1{,}7\log D = 5{,}3 - \log N$

$\log D = 3{,}12 - 0{,}59\log N$

$D = 10^{3{,}12 - 0{,}59\log N}$

$D = 10^{3{,}12} \cdot 10^{-0{,}59\log N}$

$D = 1311 \cdot 10^{\log N^{-0{,}59}}$

$D = 1311 \cdot N^{-0{,}59}$

Opgave 72:

a. $v = 0{,}86\log 350000 + 0{,}04 = 4{,}81$ feet/sec$=1{,}51\ ^{m}/_{s} = 5{,}4\ ^{km}/_{u}$

b. $v = 6\ ^{km}/_{u} = 1{,}67\ ^{m}/_{s} = 5{,}31\ ^{feet}/_{s}$

$0{,}86\log p + 0{,}04 = 5{,}31$

$0{,}86\log p = 5{,}27$

$\log p = 6{,}13$

$p = 10^{6{,}13} = 1334792$

$\frac{1334792 - 1200000}{1200000} \cdot 100\% = 11{,}2\%$ meer

c. $v = 0{,}86\log p + 0{,}04$

$-0{,}86\log p = -v + 0{,}04$

$\log p = 1{,}16v - 0{,}047$

$p = 10^{1{,}16v - 0{,}047}$

$p = 10^{-0{,}047} \cdot (10^{1{,}16})^{v}$

$p = 0{,}9 \cdot 14{,}5^{v}$

Opgave 73:

a. $x = 4{,}82$

b. $x \cdot \log 3 = \log 200$

$x = \frac{\log 200}{\log 3} = 4{,}82$

Opgave 74:

a. $1{,}2^{x} = 13$

$x = {}^{1{,}2}\log 13 = \frac{\log 13}{\log 1{,}2} = 14{,}07$

b. $x^{1{,}2} = 13$

$x = \sqrt[1{,}2]{13} = 8{,}48$

c. $3 \cdot 1{,}5^x + 1 = 19$

$3 \cdot 1{,}5^x = 18$

$1{,}5^x = 6$

$x = {}^{1,5}\log 6 = \frac{\log 6}{\log 1{,}5} = 4{,}42$

d. $5x^{1,74} + 8 = 29$

$5x^{1,74} = 21$

$x^{1,74} = 4{,}2$

$x = \sqrt[1,74]{4{,}2} = 2{,}28$

e. ${}^{3}\log 4x = 1{,}7$

$4x = 3^{1,7} = 6{,}47$

$x = 1{,}62$

f. $2 \cdot 1{,}7^{3x-1} = 46$

$1{,}7^{3x-1} = 23$

$3x - 1 = {}^{1,7}\log 23 = \frac{\log 23}{\log 1{,}7} = 5{,}91$

$3x = 6{,}91$

$x = 2{,}30$

Opgave 75:

a. $y = 3 \cdot 5^x$

$\log(3 \cdot 5^x) = \log y$

$\log 3 + \log 5^x = \log y$

$0{,}48 + x \cdot \log 5 = \log y$

$0{,}70x = -0{,}48 + \log y$

$x = -0{,}68 + 1{,}43 \cdot \log y$

b. $y = 7 \cdot 1{,}6^x$

$\log(7 \cdot 1{,}6^x) = \log y$

$\log 7 + \log 1{,}6^x = \log y$

$0{,}85 + x \cdot \log 1{,}6 = \log y$

$0{,}20x = -0{,}85 + \log y$

$x = -4{,}14 + 4{,}90 \cdot \log y$

c. $y = 1{,}8 \cdot 1{,}3^{2x-5}$

$\log(1{,}8 \cdot 1{,}3^{2x-5}) = \log y$

$\log 1{,}8 + \log 1{,}3^{2x-5} = \log y$

$0{,}26 + (2x - 5) \cdot \log 1{,}3 = \log y$

$(2x - 5) \cdot 0{,}11 = -0{,}26 + \log y$

$2x - 5 = -2{,}24 + 8{,}78 \cdot \log y$

$2x = 2{,}76 + 8{,}78 \cdot \log y$

$x = 1{,}38 + 4{,}39 \cdot \log y$

Opgave 76:

a. $P = 25 \cdot 3^{2q+4}$

$\log(25 \cdot 3^{2q+4}) = \log P$

$\log 25 + \log 3^{2q+4} = \log P$

$1{,}40 + (2q+4) \cdot \log 3 = \log P$

$(2q+4) \cdot 0{,}48 = -1{,}40 + \log P$

$2q + 4 = -2{,}93 + 2{,}10 \cdot \log P$

$2q = -6{,}93 + 2{,}10 \cdot \log P$

$q = -3{,}46 + 1{,}05 \cdot \log P$

b. $P \cdot 1{,}5^{q+2} = 18$

$\log(P \cdot 1{,}5^{q+2}) = \log 18$

$\log P + \log 1{,}5^{q+2} = 1{,}26$

$\log 1{,}5^{q+2} = 1{,}26 - \log P$

$(q+2) \cdot \log 1{,}5 = 1{,}26 - \log P$

$(q+2) \cdot 0{,}18 = 1{,}26 - \log P$

$q + 2 = 7{,}13 - 5{,}68 \cdot \log P$

$q = 5{,}13 - 5{,}68 \cdot \log P$

c. $P = 27q^{1{,}8}$

$q^{1{,}8} = 0{,}04P$

$q = (0{,}04P)^{\frac{1}{1{,}8}} = (0{,}04P)^{0{,}56} = 0{,}04^{0{,}56} \cdot P^{0{,}56} = 0{,}16P^{0{,}56}$

d. $P = 35 + 4{,}2 \log q$

$4{,}2 \log q = P - 35$

$\log q = 0{,}24P - 8{,}33$

$q = 10^{0{,}24P - 8{,}33} = 10^{-8{,}33} \cdot 10^{0{,}24P} = 4{,}6 \cdot 10^{-9} \cdot (10^{0{,}24})^P$

$q = 4{,}6 \cdot 10^{-9} \cdot 1{,}73^P$