

## 12.2 De afgeleide van $y = ax^n$

### Opgave 14:

- a.  $\frac{10}{x^3} = 10x^{-3}$
- b.  $8 \cdot \sqrt[3]{x} = 8x^{\frac{1}{3}}$
- c.  $\frac{2}{x} = 2x^{-1}$
- d.  $\frac{7}{\sqrt{x}} = \frac{7}{x^{\frac{1}{2}}} = 7x^{-\frac{1}{2}}$
- e.  $4x\sqrt{x} = 4x^1 \cdot x^{\frac{1}{2}} = 4x^{\frac{3}{2}}$
- f.  $\frac{3}{4x^2\sqrt{x}} = \frac{3}{4x^2 \cdot x^{\frac{1}{2}}} = \frac{3}{4x^{\frac{5}{2}}} = \frac{3}{4}x^{-\frac{5}{2}}$

### Opgave 15:

- a.  $f(x) = \frac{6}{x^3} + 6x^3 = 6x^{-3} + 6x^3$   
 $f'(x) = -18x^{-4} + 18x^2 = -\frac{18}{x^4} + 18x^2$
- b.  $g(x) = 3x^6 - \frac{4}{x} = 3x^6 - 4x^{-1}$   
 $g'(x) = 18x^5 + 4x^{-2} = 18x^5 + \frac{4}{x^2}$
- c.  $k(q) = \frac{1}{q} = q^{-1}$   
 $k'(q) = -q^{-2} = -\frac{1}{q^2}$
- d.  $l(p) = \frac{2}{p} + \frac{1}{2p^2} = 2p^{-1} + \frac{1}{2}p^{-2}$   
 $l'(p) = -2p^{-2} - p^{-3} = -\frac{1}{p^2} - \frac{1}{p^3}$
- e.  $h(x) = 3x\sqrt{x} - 2\sqrt{x} = 3x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$   
 $h'(x) = 4\frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 4\frac{1}{2}\sqrt{x} - \frac{1}{x^{\frac{1}{2}}} = 4\frac{1}{2}\sqrt{x} - \frac{1}{\sqrt{x}}$
- f.  $m(t) = 12t \cdot \sqrt[4]{t} = 12t^{\frac{5}{4}}$   
 $m'(t) = 15t^{\frac{1}{4}} = 15 \cdot \sqrt[4]{t}$

### Opgave 16:

- a.  $\frac{d}{dx}(x + \frac{4}{\sqrt{x}}) = \frac{d}{dx}(x + 4x^{-\frac{1}{2}}) = 1 - 2x^{-\frac{3}{2}} = 1 - \frac{2}{x\sqrt{x}}$
- b.  $\frac{d}{dx}(3x^2 \cdot \sqrt{x}) = \frac{d}{dx}(3x^{\frac{5}{2}}) = 7\frac{1}{2}x^{\frac{3}{2}} = 7\frac{1}{2}x\sqrt{x}$

- c.  $\frac{d}{dx}(3x - x^{-1,65}) = 3 + 1,65x^{-2,65}$
- d.  $\frac{d}{dx}(10 \cdot \sqrt[5]{x} + \frac{5}{x}) = \frac{d}{dx}(10x^{\frac{1}{5}} + 5x^{-1}) = 2x^{-\frac{4}{5}} - 5x^{-2} = \frac{2}{\sqrt[5]{x^4}} - \frac{5}{x^2}$
- e.  $\frac{d}{dx}(\frac{2}{5x} + \frac{5x}{2}) = \frac{d}{dx}(\frac{2}{5}x^{-1} + 2\frac{1}{2}x) = -\frac{2}{5}x^{-2} + 2\frac{1}{2} = -\frac{2}{5x^2} + 2\frac{1}{2}$
- f.  $\frac{d}{dx}((1+x)\sqrt{x}) = \frac{d}{dx}(\sqrt{x} + x\sqrt{x}) = \frac{d}{dx}(x^{\frac{1}{2}} + x^{\frac{3}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} + 1\frac{1}{2}x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}} + 1\frac{1}{2}\sqrt{x}$

### **Opgave 17:**

- a.  $y_A = \frac{1}{2}$   
 $y = \frac{2}{x} = 2x^{-1}$   
 $y' = -2x^{-2} = -\frac{2}{x^2}$   
 $y'(4) = -\frac{1}{8}$
- b.  $y'(1) = -2$
- c.  $y'(5) = -\frac{2}{25}$

### **Opgave 18:**

- a.  $10 \cdot \sqrt{x} - 5x = 0$   
 $10 \cdot \sqrt{x} = 5x$   
 $100x = 25x^2$   
 $100x - 25x^2 = 0$   
 $25x(4 - x) = 0$   
 $x = 0 \quad \vee \quad x = 4$   
 $f'(x) = 5x^{-\frac{1}{2}} - 5 = \frac{5}{\sqrt{x}} - 5$   
 $f'(4) = -2\frac{1}{2}$
- b.  $f'(\frac{1}{4}) = \frac{5}{\sqrt{\frac{1}{4}}} - 5 = \frac{5}{\frac{1}{2}} - 5 = 10 - 5 = 5$
- c.  $f'(1) = \frac{5}{\sqrt{1}} - 5 = 5 - 5 = 0$  dus de raaklijn loopt horizontaal

### **Opgave 19:**

- a.  $y = \frac{2}{x} + 2x = 2x^{-1} + 2x$   
 $y'(x) = -2x^{-2} + 2 = -\frac{2}{x^2} + 2$   
 $y'(4) = -\frac{2}{16} + 2 = 1\frac{7}{8}$
- b.  $y'(0,5) = -\frac{2}{0,25} + 2 = -8 + 2 = -6 < 0$
- c.  $y'(1) = -\frac{2}{1} + 2 = 0$  dus de raaklijn loopt horizontaal

**Opgave 20:**

a.  $f(x) = x + \frac{1}{x} = x + x^{-1}$

$$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2} = 0$$

$$1 = \frac{1}{x^2}$$

$$x^2 = 1$$

$$x = 1 \quad \vee \quad x = -1$$

$$\max f(-1) = -2$$

$$\min f(1) = 2$$

b.  $y_A = 4\frac{1}{4}$

$$f'(4) = \frac{15}{16}$$

$$k: y = \frac{15}{16}x + b \text{ door } (4, 4\frac{1}{4})$$

$$4\frac{1}{4} = \frac{15}{16} \cdot 4 + b$$

$$b = \frac{1}{2}$$

$$k: y = \frac{15}{16}x + \frac{1}{2}$$

c.  $f'(0,5) = 1 - \frac{1}{0,25} = 1 - 4 = -3$

d.  $f'(0,2) = 1 - \frac{1}{0,04} = 1 - 25 = -24 < 0$

e.  $a = 2 \quad \vee \quad a = -2$

**Opgave 21:**

a.  $y = 8 - \frac{16}{x} + \frac{24}{x^2} = 8 - 16x^{-1} + 24x^{-2}$

$$y' = 16x^{-2} - 48x^{-3} = \frac{16}{x^2} - \frac{48}{x^3} = 0$$

$$\frac{16}{x^2} = \frac{48}{x^3}$$

$$16x^3 = 48x^2$$

$$16x^3 - 48x^2 = 0$$

$$16x^2(x - 3) = 0$$

$$x = 0 \quad \vee \quad x = 3$$

$$\text{k.n.} \quad y = 5\frac{1}{3}$$

$$\text{dus } A(3, 5\frac{1}{3})$$

b.  $y'(-2) = 10$

c.  $\frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} (16x^{-2} - 48x^{-3}) = -32x^{-3} + 144x^{-4} = -\frac{32}{x^3} + \frac{144}{x^4}$

$$-\frac{32}{x^3} + \frac{144}{x^4} = 0$$

$$\frac{144}{x^4} = \frac{32}{x^3}$$

$$32x^4 = 144x^3$$

$$32x^4 - 144x^3 = 0$$

$$32x^3(x - 4\frac{1}{2}) = 0$$

$$x = 0 \quad \vee \quad x = 4\frac{1}{2}$$

$$\text{k.n.} \quad \text{dus } x = 4\frac{1}{2}$$

- d. de snelheid waarmee  $y$  verandert wordt gegeven door  $y'$  ofwel  $\frac{dy}{dx}$

$$\text{dus de snelheid is maximaal als } \left(\frac{dy}{dx}\right)' = 0$$

$$\text{dat is voor } x = 4\frac{1}{2}, \text{ dan } y = 5,63$$

$$\text{dus } B(4,5; 5,63)$$

- e. de grafiek moet 3 naar links verschoven worden, dus  $a = -3$

### **Opgave 22:**

a.  $y_A = 6,19$

$$y' = 3,6x^{-0,7} - 3$$

$$y'(4) = -1,64$$

b.  $y'(0,3) = 5,36$

c.  $3,6x^{-0,7} - 3 = 0$

$$3,6x^{-0,7} = 3$$

$$x^{-0,7} = \frac{5}{6}$$

$$x = \sqrt[0,7]{\frac{5}{6}} = 1,30$$

### **Opgave 23:**

a.  $R' = 1,6q^{-0,6} - 1$

$$R'(0,5) = 1,43 \text{ dus } \text{€ } 1,43 \text{ per stuk}$$

b.  $R'(4,5) = -0,35 \text{ dus } \text{€ } 0,35 \text{ per stuk}$

c.  $1,6q^{-0,6} - 1 = 0$

$$1,6q^{-0,6} = 1$$

$$q^{-0,6} = 0,625$$

$$q = 2,19$$

$$R(2,19) = 3,283 \text{ dus } \text{€ } 3283,-$$

### **Opgave 24:**

a.  $B = 4\sqrt{t} - t = 4t^{\frac{1}{2}} - t$

$$B' = 2t^{-\frac{1}{2}} - 1 = \frac{2}{\sqrt{t}} - 1$$

$$B'(3) = 0,15 \text{ dus } 0,15 \text{ dm}^2 \text{ ofwel } 15 \text{ cm}^2 \text{ per maand}$$

b.  $B'(4) = \frac{2}{\sqrt{4}} - 1 = 1 - 1 = 0$

$$B(4) = 4 \text{ dus } 4 \text{ dm}^2$$

**Opgave 25:**

a.  $t = 8$  geeft  $q = 25,373$  dus 25373

b.  $q' = 6 - 1,5t^{0,5}$

$t = 20$  geeft  $q' = -0,708$  dus een afname van 708 stuks per dag

c.  $t = 4$  geeft  $q = 16$

$t = 16$  geeft  $q = 32$

dus  $\frac{32-16}{16} \cdot 100\% = 100\%$  meer

d. als  $t = 36$  dan  $q = 0$

e.  $6 - 1,5t^{0,5} = 0$

$-1,5t^{0,5} = -6$

$t^{0,5} = 4$

$t = 16$  dus op 28 mei was de verkoop maximaal

$q = 32$  dus 32000 stuks