10.5 Logaritmische functies

Opgave 54:

$f(3) = 8$

$f(4) = 16$

$f(-1) = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{2}$

Opgave 55:

a. $f^{inv}(16) = 4$ want $2^4 = 16$

b. $f^{inv}(4) = 2$ want $2^2 = 4$

c. $f^{inv}(1) = 0$ want $2^0 = 1$

d. $f^{inv}(\frac{1}{2}) = -1$ want $2^{-1} = \frac{1}{2}$

Opgave 56:

a. $g^{inv}(9) = 2$ want $3^2 = 9$

b. $g^{inv}(81) = 4$ want $3^4 = 81$

c. $g^{inv}(1) = 0$ want $3^0 = 1$

d. $g^{inv}(\frac{1}{3}) = -1$ want $3^{-1} = \frac{1}{3}$

Opgave 57:

a. ${}^2\log 32 = 5$ want $2^5 = 32$

b. ${}^3\log \frac{1}{3} = -1$ want $3^{-1} = \frac{1}{3}$

c. ${}^5\log 25 = 2$ want $5^2 = 25$

d. ${}^6\log \sqrt{6} = \frac{1}{2}$ want $6^{\frac{1}{2}} = \sqrt{6}$

Opgave 58:

a. ${}^5\log 125 = {}^5\log 5^3 = 3$

b. ${}^{10}\log \frac{1}{10} = {}^{10}\log 10^{-1} = -1$

c. ${}^2\log 4 = {}^2\log 2^2 = 2$

d. ${}^7\log 49 = {}^7\log 7^2 = 2$

e. ${}^2\log \sqrt{2} = {}^2\log 2^{\frac{1}{2}} = \frac{1}{2}$

f. ${}^3\log 27 = {}^3\log 3^3 = 3$

g. ${}^2\log \frac{1}{16} = {}^2\log \frac{1}{2^4} = {}^2\log 2^{-4} = -4$

h. ${}^4\log \frac{1}{4} = {}^4\log 4^{-1} = -1$

i. ${}^5\log 5 = {}^5\log 5^1 = 1$

j. ${}^6\log 1 = {}^6\log 6^0 = 1$

k. ${}^7\log \sqrt{7} = {}^7\log 7^{\frac{1}{2}} = \frac{1}{2}$

l. ${}^2\log \frac{1}{4} = {}^2\log \frac{1}{2^2} = {}^2\log 2^{-2} = -2$

Opgave 60:

a. ${}^2\log 64\sqrt{2} = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log 2^{6\frac{1}{2}} = 6\frac{1}{2}$

b. ${}^3\log \frac{1}{9}\sqrt{3} = {}^3\log(\frac{1}{3^2} \cdot 3^{\frac{1}{2}}) = {}^3\log 3^{-1\frac{1}{2}} = -1\frac{1}{2}$

c. ${}^3\log 3^{2,76} = 2,76$

d. ${}^5\log \frac{1}{125} = {}^5\log \frac{1}{5^3} = {}^5\log 5^{-3} = -3$

e. ${}^{\frac{1}{2}}\log \frac{1}{4} = {}^{\frac{1}{2}}\log(\frac{1}{2})^2 = 2$

f. ${}^{\sqrt{5}}\log 5 = {}^{\sqrt{5}}\log(\sqrt{5})^2 = 2$

g. ${}^2\log \frac{1}{32}\sqrt[3]{2} = {}^2\log(\frac{1}{2^5} \cdot 2^{\frac{1}{3}}) = {}^2\log 2^{-4\frac{2}{3}} = -4\frac{2}{3}$

h. ${}^4\log 1 = {}^4\log 4^0 = 0$

i. ${}^3\log \sqrt[5]{3^2} = {}^3\log 3^{\frac{2}{5}} = \frac{2}{5}$

j. ${}^5\log 5^{-6\frac{1}{2}} = -6\frac{1}{2}$

k. ${}^{\frac{1}{3}}\log \frac{1}{27} = {}^{\frac{1}{3}}\log(\frac{1}{3})^3 = 3$

l. ${}^{10}\log 10000 = {}^{10}\log 10^4 = 4$

Opgave 61:

$f(\frac{1}{8}) = {}^2\log \frac{1}{8} = {}^2\log \frac{1}{2^3} = {}^2\log 2^{-3} = -3$

$f(4\sqrt{2}) = {}^2\log 4\sqrt{2} = {}^2\log(2^2 \cdot 2^{\frac{1}{2}}) = {}^2\log 2^{2\frac{1}{2}} = 2\frac{1}{2}$

$f(\sqrt[5]{4}) = {}^2\log \sqrt[5]{4} = {}^2\log \sqrt[5]{2^2} = {}^2\log 2^{\frac{2}{5}} = \frac{2}{5}$

$f(1) = {}^2\log 1 = {}^2\log 2^0 = 0$

Opgave 62:

a. ${}^3\log(x+2) = 2$

$x + 2 = 3^2$

$x + 2 = 9$

$x = 7$

b. $1 + {}^{\frac{1}{2}}\log x = 4$

${}^{\frac{1}{2}}\log x = 3$

$x = (\frac{1}{2})^3 = \frac{1}{8}$

c. ${}^3\log(2x+1) = 4$

$2x + 1 = 3^4$

$2x + 1 = 81$

$2x = 80$

$x = 40$

d. $5 + {}^4\log x = 3$

${}^4\log x = -2$

$x = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

e. ${}^{\frac{1}{2}}\log(x-1) = 3$

$x-1=(\frac{1}{2})^3$

$x-1=\frac{1}{8}$

$x=1\frac{1}{8}$

f. ${}^2\log(x^2-4)=5$

$x^2-4=2^5$

$x^2-4=32$

$x^2=36$

$x=6 \quad \vee \quad x=-6$

Opgave 63:

a. $4\cdot {}^3\log x=2$

${}^3\log x=\frac{1}{2}$

$x=3^{\frac{1}{2}}=\sqrt{3}$

b. ${}^3\log(4x-1)=-2$

$4x-1=3^{-2}$

$4x-1=\frac{1}{3^2}$

$4x-1=\frac{1}{9}$

$4x=1\frac{1}{9}$

$x=\frac{10}{36}=\frac{5}{18}$

c. $3+{}^2\log x=-1$

${}^2\log x=-4$

$x=2^{-4}=\frac{1}{2^4}=\frac{1}{16}$

d. ${}^5\log(3x+2)=1$

$3x+2=5^1$

$3x+2=5$

$3x=3$

$x=1$

e. ${}^3\log(0{,}4x-5)=2$

$0{,}4x-5=3^2$

$0{,}4x-5=9$

$0{,}4x=14$

$x=35$

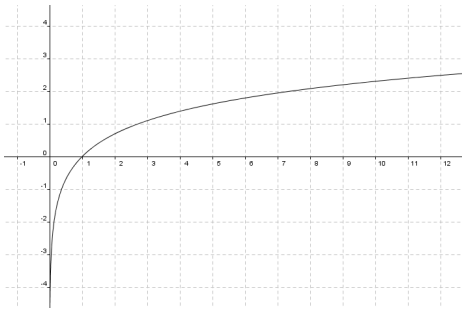
f. $4+2\cdot {}^2\log x=7$

$2\cdot {}^2\log x=3$

${}^2\log x=1\frac{1}{2}$

$x=2^{1\frac{1}{2}}=2^1\cdot 2^{\frac{1}{2}}=2\sqrt{2}$

Opgave 64:

a. 

b. $f(0{,}01) = -2$
$f(0{,}001) = -3$
$f(0{,}0000001) = -7$
als x steeds dichter naar 0 gaat dan geldt: $f(x) \to -\infty$

c. V.A.: $x = 0$

Opgave 65:

a. ${}^{3}\log 5 = \dfrac{\log 5}{\log 3} = 1{,}46$

b. ${}^{\frac{1}{7}}\log 18 = \dfrac{\log 18}{\log \frac{1}{7}} = -1{,}49$

c. $\dfrac{14}{{}^{2}\log 20 - {}^{2}\log 6} = \dfrac{14}{\frac{\log 20}{\log 2} - \frac{\log 6}{\log 2}} = 8{,}06$

d. ${}^{\frac{1}{3}}\log 10 + \log \tfrac{1}{3} = \dfrac{\log 10}{\log \frac{1}{3}} + \dfrac{\log \frac{1}{3}}{\log 10} = -2{,}57$

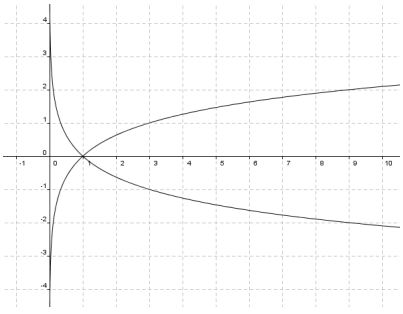
e. $3 \cdot {}^{2}\log 7 = 3 \cdot \dfrac{\log 7}{\log 2} = 8{,}42$

f. $\dfrac{5}{{}^{4}\log 12} = \dfrac{5}{\frac{\log 12}{\log 4}} = 2{,}79$

Opgave 66:

a.

x	0,1	0,5	1	1,5	3	9
$f(x)$	$-2{,}1$	$-0{,}6$	0	0,4	1	2
$g(x)$	2,1	0,6	0	$-0{,}4$	-1	-2

b. 

c. S_{x-as}

d. $y_1 = \log(x)/\log(3)$ en $y_2 = 1{,}5$
intersect geeft $x = 5{,}20$
$0 < x \leq 5{,}20$

Opgave 67:

a. $T(0,3)$

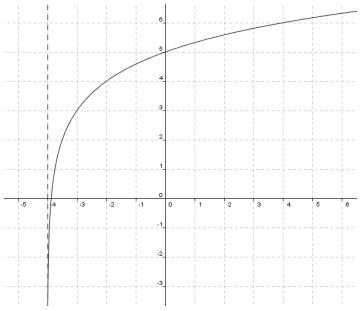
b. $T(-3,0)$

c. $V_{x-as,3}$

Opgave 68:

a. $T(-4,3)$

b. $D_f = \langle -4, \rightarrow \rangle$



Opgave 69:

a. $3x - 12 = 0$
$3x = 12$
$x = 4$ dus V.A. is $x = 4$

b. $8 - 4x = 0$
$-4x = -8$
$x = 2$ dus V.A. is $x = 2$

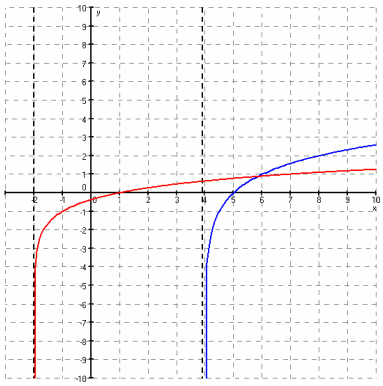
c. $8x - 10 = 0$
$8x = 10$
$x = 1{,}25$ dus V.A. is $x = 1{,}25$

d. $8 - 5x = 0$
$-5x = -8$
$x = 1{,}6$ dus V.A. is $x = 1{,}6$

Opgave 70:

a. $x + 2 > 0$
$x > -2$
$D_f = \langle -2, \rightarrow \rangle$
V.A.: $x = -2$

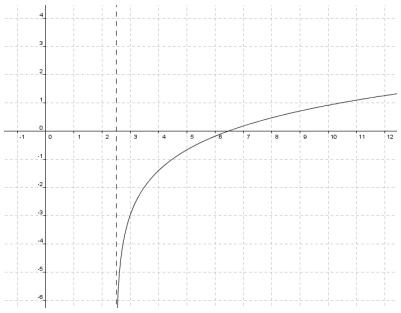
$x - 4 > 0$
$x > 4$
$D_g = \langle 4, \rightarrow \rangle$
V.A.: $x = 4$



b. $y_1 = -1 + \log(x+2)/\log(3)$
$y_2 = \log(x-4)/\log(2)$
intersect geeft: $x = 5{,}83$ en $y = 0{,}87$

c. $y_1 = -1 + \log(x+2)/\log(3)$ en $y_2 = 2{,}5$
intersect geeft $x_A = 44{,}765$
$y_2 = 2{,}5$ en $y_3 = \log(x-4)/\log(2)$
intersect geeft $x_B = 9{,}657$
$AB = x_A - x_B = 44{,}765 - 9{,}657 = 35{,}11$

Opgave 71:



a. $f(x) = -3 + {}^2\log(2x-5)$
$2x - 5 = 0$
$2x = 5$
$x = 2{,}5$
V.A.: $x = 2{,}5$

b. $f(10\frac{1}{2}) = 1$
$f(x) \le 1$

c. $-3 + {}^2\log(2x-5) = 4$
${}^2\log(2x-5) = 7$
$2x - 5 = 2^7$
$2x - 5 = 128$
$2x = 133$
$x = 66{,}5$

Opgave 72:

a. $y_1 = 10\log(x) + 120$ en $y_2 = 85$
intersect geeft $x = 3{,}2 \cdot 10^{-4}$
$I = 3{,}2 \cdot 10^{-4}$ $watt/m^2$

b. $L(10^{-7}) = 50$
$L(2 \cdot 10^{-7}) = 53$
dus er is geen sprake van een verdubbeling

c. $y_1 = 10\log(x) + 120$ en $y_2 = 50$
intersect geeft $x = 10^{-7}$
$y_1 = 10\log(x) + 120$ en $y_3 = 125$
intersect geeft $x = 3{,}2$
dus $\frac{3{,}2}{10^{-7}} = 3{,}2 \cdot 10^7 \times$ zo groot

d. $L(10^{-7}) = 50$ en $L(10^{-6}) = 60$
dus een toename van 10 dB