

5.3 Machten met gehele en gebroken exponenten.

Opgave 27:

- $x^2 \cdot x^3 = x^6$ is niet waar want $x^2 \cdot x^3 = x^5$
- $x^6 : x^2 = x^4$ is waar
- $(2x)^3 = 6x^3$ is niet waar want $(2x)^3 = 2^3 \cdot x^3 = 8x^3$
- $(x^3)^2 = x^6$ is waar

Opgave 28:

- $2a^3 \cdot 4a^7 = 8a^{10}$
- $(2a^3)^7 = 2^7 \cdot a^{21} = 128a^{21}$
- $\frac{15a^8}{3a^6} = 5a^2$
- $(3a^2b)^4 = 3^4 \cdot a^8b^4 = 81a^8b^4$
- $(5a^3)^3 \cdot 2b^7 = 5^3 \cdot a^9 \cdot 2b^7 = 125a^3 \cdot 2b^7 = 250a^9b^7$
- $21a^6 \cdot \frac{1}{3a} = \frac{21a^6}{3a} = 7a^5$
- $(-2a)^3 \cdot 3a^3 = -8a^3 \cdot 3a^3 = -24a^6$
- $(-2a)^2 + 3a^2 = 4a^2 + 3a^2 = 7a^2$
- $10a^2b \cdot \frac{2}{ab} = \frac{20a^2b}{ab} = 20a$

Opgave 29:

- $7a^3 + 5a^3 = 12a^3$
- $7a^3 - a^3 = 6a^3$
- $7a^5 : a^3 = 7a^2$
- $7a^5 \cdot 3a = 21a^6$
- $7a^3 \cdot 5a^3 = 35a^6$
- $(7a)^5 = 16807a^5$
- $(7a)^3 + 5a^3 = 343a^3 + 5a^3 = 348a^3$
- $7a \cdot 5a^3 = 35a^4$
- $(2a)^2 + (3a)^2 = 4a^2 + 9a^2 = 13a^2$
- $(-2a)^3 - 3a^3 = -8a^3 - 3a^3 = -11a^3$
- $(-3a)^2 : 2a = 9a^2 : 2a = 4\frac{1}{2}a$
- $(-3a)^2 \cdot 2a^3 = 9a^2 \cdot 2a^3 = 18a^5$

Opgave 30:

- $N = 750 \cdot 1,05^{3t+4}$
 $N = 750 \cdot 1,05^4 \cdot 1,05^{3t}$
 $N = 912 \cdot (1,05^3)^t$
 $N = 912 \cdot 1,16^t$
- $N = 18 - 3,2 \cdot 1,83^{2t+3}$

$$N = 18 - 3,2 \cdot 1,83^3 \cdot 1,83^{2t}$$

$$N = 18 - 19,61 \cdot (1,83^2)^t$$

$$N = 18 - 19,61 \cdot 3,35^t$$

c. $y = 4 \cdot (1,05x^3)^5$

$$y = 4 \cdot 1,05^5 \cdot x^{15}$$

$$y = 5,11 \cdot x^{15}$$

Opgave 31:

a. $N = 500 \cdot 1,32^{5t+2}$

$$N = 500 \cdot 1,32^2 \cdot 1,32^{5t}$$

$$N = 871,2 \cdot (1,32^5)^t$$

$$N = 871,2 \cdot 4,01^t$$

b. $P = 8,3 \cdot (5,1x^2)^3 \cdot 0,26 \cdot (x^4)^3$

$$P = 8,3 \cdot 5,1^3 \cdot x^6 \cdot 0,26 \cdot x^{12}$$

$$P = 286 \cdot x^{18}$$

Opgave 32:

a.

datum	11 mei	18 mei	25 mei	1 juni	8 juni	15 juni
t	-3	-2	-1	0	1	2
opp O	2	4	8	16	32	64

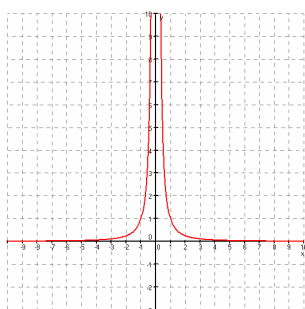
b. $2^0 = 1$

c. $2^{-1} = \frac{1}{2}$

d. $O = 2$ dus $2^{-3} = \frac{1}{8}$

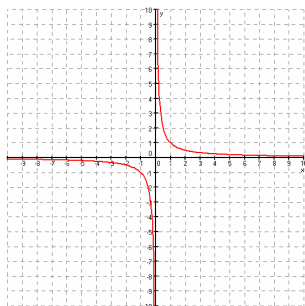
Opgave 33:

a.

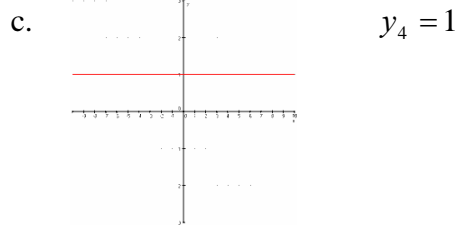


y_1 en y_2 leveren dezelfde grafiek op

b.



$$y_3 = \frac{1}{x}$$



Opgave 34:

a. $\frac{1}{a^2} = a^{-2}$

b. $a^4 \cdot \frac{1}{a^6} = a^4 \cdot a^{-6} = a^{-2}$

c. $a^n : \frac{1}{a^4} = a^n : a^{-4} = \frac{a^n}{a^{-4}} = a^{n+4}$

d. $a^8 : a^0 = a^8 : 1 = a^8$

e. $(a^3)^{-2} = a^{-6}$

f. $\frac{a}{a^{12}} = a^{-11}$

g. $\frac{1}{a^5} : a = a^{-5} : a = \frac{a^{-5}}{a} = a^{-6}$

h. $\frac{1}{a^n} : a^{-3} = a^{-n} : a^{-3} = \frac{a^{-n}}{a^{-3}} = a^{-n+3}$

i. $\frac{1}{a^8} \cdot (a^3)^n = a^{-8} \cdot a^{3n} = a^{3n-8}$

Opgave 35:

a. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

b. $(\frac{1}{3})^{-2} = (3^{-1})^{-2} = 3^2 = 9$

c. $3 \cdot 5^{-2} = 3 \cdot \frac{1}{5^2} = 3 \cdot \frac{1}{25} = \frac{3}{25}$

d. $(\frac{2}{5})^{-2} = \frac{2^{-2}}{5^{-2}} = \frac{5^2}{2^2} = \frac{25}{4} = 6\frac{1}{4}$

e. $4 \cdot 10^{-3} = 4 \cdot \frac{1}{10^3} = 4 \cdot \frac{1}{1000} = \frac{4}{1000} = \frac{1}{250}$

f. $\frac{1}{2} : 6^{-2} = \frac{1}{2} : \frac{1}{6^2} = \frac{1}{2} : \frac{1}{36} = \frac{1}{2} \cdot \frac{36}{1} = \frac{36}{2} = 18$

Opgave 36:

a. $6a^{-5} \cdot b^3 = \frac{6}{a^5} \cdot b^3 = \frac{6b^3}{a^5}$

b. $\frac{1}{3}a^{-3} = \frac{1}{3} \cdot \frac{1}{a^3} = \frac{1}{3a^3}$

c. $5a^{-4} \cdot b^2 = \frac{5}{a^4} \cdot b^2 = \frac{5b^2}{a^4}$

d. $\frac{3}{5}a^{-4} = \frac{3}{5} \cdot \frac{1}{a^4} = \frac{3}{5a^4}$

e. $(\frac{1}{2}a)^{-3} = (2^{-1} \cdot a)^{-3} = 2^3 \cdot a^{-3} = 8 \cdot \frac{1}{a^3} = \frac{8}{a^3}$

f. $\frac{1}{6}a^{-2} \cdot b^4 = \frac{1}{6} \cdot \frac{1}{a^2} \cdot b^4 = \frac{b^4}{6a^2}$

g. $-4 \cdot (3a)^{-2} = -4 \cdot \frac{1}{(3a)^2} = -4 \cdot \frac{1}{9a^2} = \frac{-4}{9a^2}$

h. $(3a)^{-2} \cdot b^{-3} = \frac{1}{(3a)^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2b^3}$

i. $\frac{3}{8}a^{-1} \cdot b = \frac{3}{8} \cdot \frac{1}{a} \cdot b = \frac{3b}{8a}$

Opgave 37:

a. $y = (\frac{1}{3}x^2)^{-1} \cdot x^4$
 $y = (3^{-1} \cdot x^2)^{-1} \cdot x^4$
 $y = 3 \cdot x^{-2} \cdot x^4$
 $y = 3x^2$

b. $y = 75 \cdot (5x)^{-2} \cdot 3x^{12}$
 $y = 75 \cdot 5^{-2} \cdot x^{-2} \cdot 3x^{12}$
 $y = 75 \cdot \frac{1}{5^2} \cdot x^{-2} \cdot 3x^{12}$
 $y = 75 \cdot \frac{1}{25} \cdot x^{-2} \cdot 3x^{12}$
 $y = 9x^{10}$

c. $y = \frac{5}{x^2} \cdot (3x^{-2})^3$
 $y = 5x^{-2} \cdot 3^3 \cdot x^{-6}$
 $y = 5x^{-2} \cdot 27x^{-6}$
 $y = 135x^{-8}$

d. $y = 50 \cdot 2^{3x-1}$
 $y = 50 \cdot 2^{3x} \cdot 2^{-1}$
 $y = 50 \cdot (2^3)^x \cdot \frac{1}{2}$
 $y = 25 \cdot 8^x$

e. $y = 275 \cdot 5^{-2x}$
 $y = 275 \cdot (5^{-2})^x$
 $y = 275 \cdot (\frac{1}{5^2})^x$
 $y = 275 \cdot (\frac{1}{25})^x$

f. $y = 5000 \cdot 10^{-x-3}$
 $y = 5000 \cdot 10^{-x} \cdot 10^{-3}$
 $y = 5000 \cdot (10^{-1})^x \cdot \frac{1}{10^3}$
 $y = 5000 \cdot (\frac{1}{10})^x \cdot \frac{1}{1000}$

$$y = 5 \cdot \left(\frac{1}{10}\right)^x$$

Opgave 38:

- a. klopt, want $\left(2^{\frac{1}{7}}\right)^7 = 2^{\frac{1}{7} \cdot 7} = 2^1 = 2$
 b. klopt
 c. als $\left(2^{\frac{1}{7}}\right)^7 = 2$ en $\left(\sqrt[7]{2}\right)^7 = 2$ dan moet gelden $\left(2^{\frac{1}{7}}\right)^7 = \left(\sqrt[7]{2}\right)^7$ dus $2^{\frac{1}{7}} = \sqrt[7]{2}$
 d. klopt

Opgave 39:

- a. $5a^{\frac{1}{3}} = 5 \cdot \sqrt[3]{a}$
 b. $\frac{1}{2}a^{-\frac{1}{4}}b = \frac{1}{2} \cdot \frac{1}{a^{\frac{1}{4}}} \cdot b = \frac{1}{2} \cdot \frac{1}{\sqrt[4]{a}} \cdot b = \frac{b}{2 \cdot \sqrt[4]{a}}$
 c. $3a^{-\frac{2}{3}} = 3 \cdot \frac{1}{a^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{a^2}}$
 d. $\frac{2}{3} \cdot a^{-3} \cdot b^{\frac{1}{3}} = \frac{2}{3} \cdot \frac{1}{a^3} \cdot \sqrt[3]{b} = \frac{2 \cdot \sqrt[3]{b}}{3a^3}$
 e. $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{3}} = \frac{1}{5} \cdot \frac{1}{a^{\frac{1}{2}}} \cdot \sqrt[3]{b} = \frac{\sqrt[3]{b}}{5 \cdot \sqrt{a}}$
 f. $(5a)^{-\frac{1}{2}} = \frac{1}{(5a)^{\frac{1}{2}}} = \frac{1}{\sqrt{5a}}$

Opgave 40:

- a. $a \cdot \sqrt[3]{a} = a^1 \cdot a^{\frac{1}{3}} = a^{\frac{4}{3}}$
 b. $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$
 c. $\frac{1}{a} = a^{-1}$
 d. $\frac{1}{a^3} = a^{-3}$
 e. $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{\frac{5}{2}}$
 f. $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$
 g. $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$
 h. $a^4 \cdot \sqrt[3]{a} = a^4 \cdot a^{\frac{1}{3}} = a^{\frac{13}{3}}$
 i. $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{\frac{8}{3}}$

Opgave 41:

- a. $\frac{x^6}{x^2 \cdot \sqrt{x}} = \frac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \frac{x^6}{x^{\frac{5}{2}}} = x^{\frac{7}{2}}$
 b. $x \cdot \sqrt[7]{x^3} = x^1 \cdot x^{\frac{3}{7}} = x^{\frac{10}{7}}$

c. $\frac{x}{\sqrt[5]{x}} = \frac{x^1}{x^{\frac{1}{5}}} = x^{\frac{4}{5}}$

d. $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$

e. $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{-\frac{1}{6}}$

f. $\frac{1}{x^2} : \sqrt{x} = x^{-2} : x^{\frac{1}{2}} = \frac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2\frac{1}{2}}$

g. $x^2 \cdot \frac{1}{x^3} = \frac{x^2}{x^3} = x^{-1}$

h. $x^5 \cdot \sqrt[3]{x^6} = x^5 \cdot x^{\frac{6}{3}} = x^5 \cdot x^2 = x^7$

i. $\frac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \frac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \frac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{-1\frac{1}{20}}$

Opgave 42:

a. $y = \frac{5}{x\sqrt{x}}$
 $y = \frac{5}{x^1 \cdot x^{\frac{1}{2}}}$
 $y = \frac{5}{x^{1\frac{1}{2}}}$
 $y = 5x^{-1\frac{1}{2}}$

b. $y = 5x \cdot \sqrt[3]{x^2}$
 $y = 5x \cdot x^{\frac{2}{3}}$
 $y = 5x^{1\frac{2}{3}}$

c. $y = \frac{5}{x^3} \cdot 2\sqrt{x}$
 $y = 5x^{-3} \cdot 2x^{\frac{1}{2}}$
 $y = 10x^{-2\frac{1}{2}}$

d. $y = 3 \cdot \sqrt[4]{x^3}$
 $y = 3x^{\frac{3}{4}}$

e. $y = 5x^{-0,2} \cdot x^{1,3}$
 $y = 5x^{1,1}$

f. $y = \frac{50x^{1,9}}{10x^{1,1}}$
 $y = 5x^{0,8}$

Opgave 43:

a. $-3\sqrt[3]{210} = -5,944$ en $(-3)\sqrt[3]{210} = 0,168$

- b. $(-3)\sqrt[5]{210}$
 c. nee
 $x^3 = -7$ geeft $x = \sqrt[3]{-7} = -1,91$
 $x^{-3} = 7$ geeft $x = \sqrt[3]{7} = 0,52$

Opgave 44:

- a. $x^{1,6} = 50$
 $x = \sqrt[1,6]{50} = 11,53$
 b. $x^{-4} = 5$
 $x = \sqrt[4]{5} = 0,67$
 c. $x^{-1,3} = 11$
 $x = \sqrt[1,3]{11} = 0,16$
 d. $x^{-1} = 21$
 $x = \sqrt[1]{21} = 0,05$
 e. $x^{0,55} = 18$
 $x = \sqrt[0,55]{18} = 191,56$
 f. $\sqrt[3]{x^2} = 28$
 $x^2 = 28^3 = 21952$
 $x = \sqrt{21952} = 148,16$

Opgave 45:

- a. $3x^{2,25} + 1 = 27$
 $3x^{2,25} = 26$
 $x^{2,25} = \frac{26}{3}$
 $x = \sqrt[2,25]{\frac{26}{3}} = 2,611$
 b. $5x^{-1,3} + 8 = 21$
 $5x^{-1,3} = 13$
 $x^{-1,3} = 2,6$
 $x = \sqrt[1,3]{2,6} = 0,480$
 c. $4x^{-1,8} + 16 = 5000$
 $4x^{-1,8} = 4984$
 $x^{-1,8} = 1246$
 $x = \sqrt[1,8]{1246} = 0,019$
 d. $8 - 3x^{1,16} = 1$
 $-3x^{1,16} = -7$
 $x^{1,16} = \frac{7}{3}$
 $x = \sqrt[1,16]{\frac{7}{3}} = 2,076$
 e. $5 \cdot \sqrt[3]{x} = 8$
 $\sqrt[3]{x} = 1,6$
 $x = 1,6^3 = 4,096$

$$\begin{aligned}
 \text{f. } & 3 \cdot \sqrt[4]{x^3} - 1 = 36 \\
 & 3 \cdot \sqrt[4]{x^3} = 37 \\
 & \sqrt[4]{x^3} = \frac{37}{3} \\
 & x^3 = \left(\frac{37}{3}\right)^4 = 23137,79 \\
 & x = \sqrt[3]{23137,79} = 28,495
 \end{aligned}$$

Opgave 46:

$$\begin{aligned}
 \text{a. } & H = 12 \cdot 10^{0,67} = 56 \text{ gram} \\
 \text{b. } & 12 \cdot G^{0,67} = 18 \\
 & G^{0,67} = 1,5 \\
 & G = \sqrt[0,67]{1,5} = 1,8 \text{ kg} \\
 \text{c. } & H = 12 \cdot 40^{0,67} = 142 \\
 & I = \frac{130}{142} = 0,91 \\
 \text{d. } & H = 12 \cdot 70^{0,67} = 206,7 \\
 & I = \frac{1650}{206,7} = 8,0 \\
 \text{e. } & \frac{1050}{H} = 1,04 \\
 & H = \frac{1050}{1,04} = 1009,6 \\
 & 12 \cdot G^{0,67} = 1009,6 \\
 & G^{0,67} = \frac{1009,6}{12} = 84,1 \\
 & G = \sqrt[0,67]{84,1} = 747 \text{ kg}
 \end{aligned}$$

Opgave 47:

$$\text{a. } P = 800 \cdot l^{-2,25} = \frac{800}{l^{2,25}}$$

dus als l groter wordt, dan wordt $l^{2,25}$ groter, dus wordt P kleiner omdat $l^{2,25}$ in de noemer staat.

Als P kleiner wordt betekent dat dat er minder organismen per km^2 leven, dus de bewering klopt.

$$\begin{aligned}
 \text{b. } & P = 800 \cdot 0,9^{-2,25} = 1014 \\
 \text{c. } & 800 \cdot l^{-2,25} = 1350 \\
 & l^{-2,25} = 1,6875 \\
 & l = \sqrt[-2,25]{1,6875} = 0,79 \text{ m dus } 79 \text{ cm} \\
 \text{d. } & P = 800 \cdot 2,15^{-2,25} = 142,9 \\
 & 142,9 \cdot 250 = 35731 \text{ kariboes} \\
 \text{e. } & \frac{160000}{5} = 32000 \\
 & 800 \cdot l^{-2,25} = 32000 \\
 & l^{-2,25} = 40 \\
 & l = \sqrt[-2,25]{40} = 0,19 \text{ m}
 \end{aligned}$$

Opgave 48:

a. $0,059 \cdot G^{0,92} = 4$

$$G^{0,92} = 67,8$$

$$G = \sqrt[0,92]{67,8} = 97,8 \text{ kg}$$

b. omnivoor: $0,059 \cdot G^{0,92} = 6$

$$G^{0,92} = 101,7$$

$$G = \sqrt[0,92]{101,7} = 152$$

carnivoor: $0,11 \cdot G^{1,36} = 6$

$$G^{1,36} = 54,5$$

$$G = \sqrt[1,36]{54,5} = 19$$

de omnivoor is het zwaarst, het verschil is $152 - 19 = 133 \text{ kg}$

c. het leefgebied is dan $4^{1,36} = 6,6 \times$ zo groot

d. $L_C^* = 0,11 \cdot (0,001 \text{ g})^{1,36} = 0,11 \cdot 0,001^{1,36} \cdot \text{g}^{1,36} = 0,000009 \text{ g}^{1,36}$

Opgave 49:

$$2,5 \cdot 16^{1,5} = 160$$

Opgave 50:

$$a \cdot 12^{-1,81} = 16$$

$$a = \frac{16}{12^{-1,81}} = 1437$$

Opgave 51:

$$a \cdot 18^{1,83} = 350$$

$$a = \frac{350}{18^{1,83}} = 1,7$$

$$y = 1,766 \cdot 25^{1,83} = 638$$

Opgave 52:

a. $a \cdot 40^{0,75} = 6700$

$$a = \frac{6700}{40^{0,75}} = 421,2$$

$$W = 421,2 \text{ m}^{0,75}$$

b. $W = 421,2 \cdot 4^{0,75} = 1191 \text{ kJ}$

c. $421,2 \text{ m}^{0,75} = 50000$

$$m^{0,75} = 118,7$$

$$m = \sqrt[0,75]{118,7} = 583 \text{ kg}$$

Opgave 53:

a. $a \cdot 30^{0,88} = 90$

$$a = \frac{90}{30^{0,88}} = 4,51$$

$$W = 4,51m^{0,88}$$

- b. $W = 4,51 \cdot 200^{0,88} = 477,6 \frac{ml}{uur}$
dus per dag: $24 \cdot 477,6 = 11463 \text{ ml} = 11,5 \text{ liter}$
- c. per uur: $\frac{1200}{24} = 50 \text{ ml}$
 $4,51m^{0,88} = 50$
 $m^{0,88} = 11,09$
 $m = \sqrt[0,88]{11,09} = 15 \text{ kg}$
- d. $W = \frac{24}{1000} \cdot 4,51m^{0,88} = 0,10824m^{0,88}$
- e. $4^{0,88} = 3,4 \times \text{zo veel}$